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Abstract

How does an increase in unionization costs, i.e. costs which arise when workers are organized by a union, affect the productivity distribution of active firms, wage inequality and welfare? In this paper, we build a model with costly, endogenous unionization, heterogeneous firms as well as free market entry/exit. If unionization costs are relatively low (high), we find that an increase in these costs decreases (increases) average productivity and welfare decreases (increases). Additionally, we find a hump-shaped relationship between unionization costs and wage inequality. These results suggest that policies aiming to reduce unionization could worsen economic performance.

Keywords: unionization costs, endogenous unionization, firm-selection, welfare, wage inequality

JEL Classification: J51, L11, L16

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1 Introduction

During the last two decades, unionization (measured by the share of union members among employees) has substantially declined in most OECD countries (see Visser, 2015). One important reason for this trend is that organizing a workforce is increasingly costly (see Economist, 2013). These unionization costs include direct costs, for example salaries and expenses of the union’s representatives, but also indirect costs, for example unpaid time which current members spend to coordinate. Additionally, the government (implicitly) influences unionization costs by setting (or changing) the legal framework under which unionization takes place. There are several examples of policy makers trying to decrease unionization by raising the legal barriers such as the Right-to-work laws in several states in the US, the ‘Tarifeinheitsgesetz’ in Germany (which only allows the largest union in a firm to set the wage) or the ban on closed shop unions in the UK in the late 1980s.

A primary motivation for implementing these policies is the notion that with higher costs, workers refrain from forming a union. Deunionization then lowers wages and improves the labor market allocation and hence welfare. This argument, however, may be misguided for at least for two reasons. First, the union will adjust its wage setting strategy in light of the new institutional framework. There may be situations in which workers refrain forming a union, but those who do not will increase wage demands to make up for the higher costs they (implicitly or explicitly) have to bear. When or whether this will happen when taking the union formation choice of workers into account remains an open question. Second, not only does the unions’ behavior change, but the behavior of active and prospective firms changes as well. If the policy, for example, leads to deunionization and lower wages, firms with less productive technologies might be induced to enter the market. This is basically the ‘Scandinavian’ argument that higher unionization drives inefficient firms out of the market, see Agell and Lommerud (1993). But it also affects the decisions of already active firms, because the composition of the industry in unionized and non-unionized firms changes. Again, the induced effect of the change in unionization costs remains, to the best of our knowledge, largely unexplored.

The aim of our paper is to analyze the allocative effects of unionization and unionization costs when taking the extensive margins of firm and union formation into account. We thereby provide a more comprehensive understanding of how policies that are motivated to decrease unionization affect the productivity distribution of firms, wage inequality and welfare.

To this end, we set up a model where firms are heterogeneous in productivity, operate under conditions of monopolistic competition and face market
entry as well as fixed production costs (see Melitz, 2003). Given the expected structure of the industry, firms not only decide on how much to produce, but also whether to enter the industry. Unions act at the firm-level as wage setting monopoly unions and face unionization costs (see Kuhn, 1998). Workers only form a union and delegate wage setting power to it if the (expected) acquired rent is at least as high as individual (=average) unionization costs. If workers refrain from forming a union, the firm pays the competitive wage.

We derive the equilibrium allocation under a set of simplifying assumptions, which helps us find closed form solutions. First, we assume ex-ante productivities to be Pareto distributed (Axtell, 2001, Helpman et al., 2004). Second, we abstract from wealth effects (following Chor, 2009 and Cole and Davies, 2011, among others). Third, we follow recent literature in assuming firm-level unions.1 Fourth, we consider monopoly unions as done by MacDonald and Robinson (1992).2 Fifth, we assume that marginal and average unionization costs are constant.3

Concerning the distribution of unionization in equilibrium, i.e. which of the active firms face wage setting and which not, we find that low-productivity firms are not unionized.4 Profits that could be captured by wage setting are too low to cover unionization costs and workers refrain from forming a union. Firms with intermediate productivity levels are unionized. When setting the wage in these firms, the union follows a limit wage strategy which implies setting a wage that leaves the firm with zero profits. This is optimal, because the unconstrained wage would lead to negative profits implying that firms exit the market, leaving the union with zero utility.5 For high-productivity firms, the union sets the utility maximizing unconstrained wage.

The equilibrium productivity distribution is driven by the firm’s decision to produce conditional on the productivity that has been drawn. As long as the firm’s profit is positive, it starts production. The zero profit condition then determines a cut-off productivity which not only determines the lower productivity bound, but also determines average productivity in the industry.

1In OECD countries, union wage bargaining occurs at several levels, including the national, sectoral and the firm level. However, there is a tendency towards more decentralized wage determination also in countries where wages traditionally are determined at the sectoral level (see, e.g., Ochel, 2005 and Gürtzgen, 2009 for Germany).
2We show that a Nash bargaining approach leads to qualitatively similar results.
3Extending our model to increasing/decreasing marginal and average unionization costs implies that we cannot find closed form solutions. The numerical solution shows, however, that our results are quite robust.
4This matches with the observation that large and productive firms are unionized (see Dinlersoz et al., 2017).
5This is in line with Freeman and Kleiner (1999) and Hirsch (2004), who find that unions generally reduce firms’ profits, but only to an extent that prevents firm failure.
We show that an increase in unionization costs has two countervailing effects on the productivity distribution in the economy. First, the union will increase its wage demands to make up for the cost increase, if this is possible without driving active firms out of the market. This will be the case for high productivity firms. These firms have then to pay higher wages, charge higher prices and cut back on production. This increases the incentive to enter the market, because competition from these high productivity firms is weakened. Simultaneously, some firms are freed from unionization, because the accruable profits in these firms is lower than the unionization costs. Workers do not form a union and the wage declines to competitive levels. This decreases the incentive to enter the market, because competition becomes more fierce.

We show that for low levels of unionization costs (which translates into a high degree of unionization), the first effect dominates and vice versa. The relation between unionization (costs) and (average) productivity is U-shaped. With unionization costs initially very low, the fraction of firms that pay the unconstrained wage is large and that of the limiting wage paying firms is low. Then a large fraction of firms face the wage increase caused by higher unionization costs, implying that the incentive for market entry dominates.

By the same token, we can show that the relation between unionization costs and wage inequality (as measured by the Gini coefficient) is hump-shaped. The intuition again is that with higher unionization costs, workers in high productivity firms enjoy wage increases, whereas those in intermediate productivity firms face wage decreases due to deunionization. The first effect dominates for low levels of unionization costs and vice versa. Concerning welfare, we show that the relation between unionization costs and welfare is U-shaped. The intuition is that first, income monotonously decreases in unionization costs. Second, if unionization costs are low, the reduced cutoff productivity implies that firms are, on average, less productive, which raises the price index and thus reduces consumption. Only if unionization costs are sufficiently high, a welfare-enhancing effect of lower unionization exists due to the increase in cutoff productivity and the reduction of the price index.

Our paper adds to the literature on unionization costs and endogenous unionization. A focus in this literature is the strategic effect in forming a union either on the side of workers in the light of free-riding (see Booth, 1985 or Booth and Chatterji, 1995), or on the side of the firm by the means of management opposition against union formation (see Corneo, 1995). We abstain from these strategic considerations to isolate the 'pure' effect of union formation on the wage structure, productivity and firm entry. In this respect, the paper by Kuhn (1988) is closest to what we consider. He analyzes a general equilibrium model of occupational choice where heterogeneous households choose to become entrepreneurs or workers. Union formation then distorts
this occupational decision. We complement his analysis by taking the various fixed-cost margins, which entrepreneurs face and which affect their market entry and production decisions, directly into account. With this, we get a more detailed picture of how unionization costs affect the productivity distribution of firms and how it changes the wage distribution as well as welfare.

The remainder of our paper is structured as follows. In section 2, we lay out the basic environment of our model and describe the timing of events. The partial equilibrium is determined in section 3, while the general equilibrium is described in section 4. The impact of unionization costs on productivity, welfare and wage inequality is investigated in section 5, where we present both an analytical and a numerical approach. In section 6, we discuss the robustness of our model. Section 7 concludes.

2 The Model

2.1 Structure of the Economy

The economy consists of two sectors. In one sector, a homogenous good $Y$ is produced and sold under conditions of perfect competition. We choose good $Y$ to be the numeraire. Production of this good is linear in labor and, due to perfect competition, the wage is given by $w_Y = 1$. In the other sector, a mass of $M$ firms produces varieties of a (horizontally) differentiated good $X$. There is monopolistic competition in the goods market and each firm produces one variety. Moreover, workers in a firm may decide to form a union, in which case the firm-level wage is unilaterally set by the union. Otherwise, firms pay the competitive wage.

There is an exogenously given mass of workers $L$ who are homogeneous and mobile across sectors. Each worker inelastically supplies labor. Workers who are not employed in the $X$-sector find a job in the $Y$-sector, so that there is no involuntary unemployment. The mass of workers employed in the $Y$-sector ($X$-sector) is denoted by $L_Y$ ($L_X$).

2.2 Consumers and Demand

Following Cole and Davies (2011) we assume the utility function of the representative consumer to be quasi-linear

$$U = \nu \ln X + Y, \quad X \equiv \left[ \int_0^M x(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}.$$  (1)

6The good $Y$ could be interpreted as leisure such that the ‘production’ of $Y$ reflects voluntary unemployment.
$X$ is a CES index of all available varieties in the differentiated good sector. The relative weight of consuming $X$ is denoted by $\nu > 0$. Varieties are consumed in quantity $x(\omega)$, where $\omega$ is the index for the firm producing this variety. The elasticity of substitution between varieties is given by $\sigma = 1/(1-\rho) > 1$.

The representative consumer maximizes utility by choosing $Y$ and $x(\omega)$ subject to $\int_0^M p(\omega)x(\omega)d\omega + Y = I$, where $p(\omega)$ and $I$ denote the price of variety $\omega$ and income, respectively. Demand for the homogeneous and the differentiated goods are given by

$$Y = I - \nu, \quad (2)$$
$$x(\omega) = \nu p(\omega)^{-\sigma} P^{\sigma-1}, \quad (3)$$

where $P$ represents the CES price index for the composite good $X$

$$P = \left[ \int_0^M p(\omega)^{-\frac{\rho}{1-\rho}} d\omega \right]^{-\frac{1-\rho}{\rho}}. \quad (4)$$

The demand for the composite is

$$X = \frac{\nu}{P}, \quad (5)$$

which shows that expenditures for the good $X$ are fixed at $\nu$ such that income affects only the consumption of the homogeneous good.

### 2.3 Firms and Production

The economy consists of an unbounded pool of potential entrants into the $X$-sector which we call entrepreneurs. Following Melitz (2003), we assume that each entrepreneur makes two decisions. First, he decides whether to enter the market and second, conditional on market entry, he decides whether to become an active firm and produce or to leave the market.

Market entry requires that an entrepreneur pays fixed costs $F_e$ which are sunk thereafter. Once $F_e$ has been paid, the entrepreneur is endowed with a productivity level $\phi$ which is drawn from a given distribution. Intuitively, $F_e$ measures (non-reversible) costs for research and development, which firms have to bear in advance of production and which determine their productivity. Let $M_e$ denote the mass of entrepreneurs which are willing to incur the fixed costs $F_e$. We assume that productivities are Pareto distributed with the distribution function

$$G(\phi) = 1 - \left( \frac{\phi_{\text{min}}}{\phi} \right)^k, \quad (6)$$
where $\phi_{\text{min}} \geq 1$ is the lower bound of the support and $k > \sigma - 1$ is the shape parameter.

After the entrepreneur is endowed with its productivity, he decides about production which requires (reversible) fixed costs $F$. These costs consists of, for example, raw materials and capital needed for production.\textsuperscript{7} The marginal firm which is indifferent between production and market exit is characterized by productivity $\phi^*$. Entrepreneurs start production if $\phi \geq \phi^*$ and leave the market otherwise. The mass of active firms is given by $M = (1 - G(\phi^*))M_e$.

In the $X$-sector, labor $l$ is the only input. The production function reads

$$x = \phi \cdot l$$

and profits are defined as

$$\pi = px - wl - F = \left(p - \frac{w}{\phi}\right)x - F,$$

where $w$ represents the wage rate which the firm has to pay to its workers.

### 2.4 Unions and Unionization Costs

We assume that initially workers are allocated between firms in the $X$-sector according to the opportunity costs of working there (which is the wage in the $Y$-sector). Workers then decide to form a union. If a union is formed, it unilaterally sets the wage on behalf of its members, the firm’s initial workforce $\tilde{l}$, while the firm retains the right to manage employment and adjusts its workforce to a level $l$ in light of the wage set by the union.\textsuperscript{8} If workers decide not to form a union, all initially employed workers remain employed and get their opportunity costs of working compensated (by paying the competitive wage).

Forming a union is costly. As argued by Voos (1983), unionization costs consist of direct costs, for example salaries and expenses of the union’s representatives, and indirect costs, for example unpaid time which (employed) members spend to coordinate. Additionally, unionization costs include institutional costs due to legal barriers to unionization, which can be influenced by policy makers. These costs are summarized in a cost function denoted by $C$. Following MacDonald and Robinson (1992), we presume that $C$ is an

\textsuperscript{7}The assumption of reversible fixed costs implies that in any stage of the game, the firm’s outside option is to exit the market without making a loss. Note that market entry costs are sunk and therefore are not relevant for the decision to leave the market.

\textsuperscript{8}This assumption is made for analytical simplicity. We show in section 6.1 that a Nash-bargaining over wages between firms and unions leads to a qualitatively similar wage path.
increasing function of employed union members \( l \). Unemployed union members do not incur any costs. Whether unionization costs are convex, linear or concave in \( l \) is, however, not clear-cut and thus an empirical question. We consider a linear cost function which allows us to find closed-form solutions.\(^9\) Specifically, we assume

\[
C(l) = c \cdot l.
\]

A worker’s expected utility in a unionized firm is given by (due to the linearity of the utility function in income)

\[
\mathbb{E}(U) = \frac{l}{l} (w - c) + \left(1 - \frac{l}{l}\right) w_Y,
\]

where the outside option of the union member is to be employed in the \( Y \)-sector and receive a wage payment of \( w_Y = 1 \). As such, we neglect the possibility that a worker might also find a job at another firm in the \( X \)-sector, which is a reasonable assumption if the initial allocation requires some sort of search or specific investment (which we do not explicitly model here).

Aggregating over all its members (at the firm-level), the union’s utility function is given by

\[
\tilde{l}\mathbb{E}(U) = l \left( w - \frac{C}{l}\right) + \tilde{l} - l, \tag{11}
\]

where we rewrite (11) for notational convenience as

\[
\tilde{l}\mathbb{E}(U) - \tilde{l} := V = l(w - 1) - cl. \tag{12}
\]

### 2.5 Timing and Solution Strategy

The timing structure of our model is as follows.

1. Each entrepreneur decides about market entry, i.e. investing \( F_e \) and drawing a productivity level. Conditional on market entry, the entrepreneur decides whether to pay \( F \) and produce or to exit the market without production.

2. Workers are allocated between active firms according to their opportunity costs. Workers decide to form a union, which then sets the wage rate on behalf of its members.

\(^9\)In section 6.2, we discuss how increasing/decreasing marginal and average unionization costs as well as a fixed cost component affect our results. Note that the latter reflects costs that are related to the initial work force \( \tilde{l} \), i.e. unemployed union members. Solving our model numerically shows that these extensions lead to qualitatively similar wage paths.
3. Each firm has the opportunity to set the profit maximizing price and to readjust production and employment. If it chooses to stop production, it can also regain its fixed costs $F$.

To determine the partial equilibrium, we consider a firm with productivity $\phi$ producing the variety $\omega$ and solve the aforementioned game by backwards induction. At this stage, we treat the variables $P$, $M$, $M_e$, $I$ and $\phi^*$ as exogenously given. To compute the general equilibrium, we endogenize these measures.

### 3 Partial Equilibrium

#### 3.1 The Firm’s Choices

Profits are maximized by choosing the price $p$ subject to the demand function (3) and the wage $w$ which has been determined at stage 2. This yields the profit-maximizing price:

$$p(\phi, w) = \frac{1}{\rho \phi} w,$$

which is a constant mark-up $1/\rho$ (due to CES preferences) over firm-specific variable costs.

Plugging (13) into the demand function (3) yields firm’s output: $x(\phi, w) = \nu p(\phi, w)^{-\sigma} P^{\sigma-1}$. Employment is $l(\phi, w) = x(\phi, w)/\phi = \nu \phi^{\sigma-1} p(\phi, w)^{-\sigma} P^{\sigma-1}$. Plugging (3) into the definition of the firm’s revenues, $r = px$, and observing (13) yields the profit-maximizing revenue

$$r(\phi, w) = \nu p(\phi, w)^{-\sigma} P^{\sigma-1}.$$

Substituting (13) and the profit-maximizing output into (8) leads to the profit function

$$\pi(\phi, w) = (1 - \rho) r(\phi, w) - F.$$

We define a feasible wage rate $\tilde{w}(\phi)$ at which (maximized) profits of a firm with productivity $\phi$ are zero, $\pi(\phi, \tilde{w}(\phi)) = 0$. If the wage rate was higher than $\tilde{w}(\phi)$, the firm would face a loss and would choose to exit the market. The feasible wage is

$$\tilde{w}(\phi) = \rho \phi P \left( \frac{\sigma F}{\nu} \right)^{-\frac{1}{\sigma-1}},$$

which shows that $\tilde{w}(\phi)$ is increasing in $\phi$ because more productive firms survive at higher wage costs.
3.2 The Union’s Choices

Suppose that the workers have formed a union. Then, the wage \( w \) is chosen to maximize utility (12) subject to the firm’s employment choice. Additionally, the union has to take into account that the wage must not exceed the firm’s feasible wage \( \tilde{w}(\phi) \) because a firm’s market exit would leave the union with zero utility. The union’s optimization problem reads

\[
\max_w V = l(w - 1) - cl \\
\text{s.t. } l = l(\phi, w) \text{ and } w \leq \tilde{w}(\phi).
\]

The Lagrange function of that problem is given by

\[
L = l(\phi, w)(w - 1) - cl(\phi, w) + \eta(\tilde{w}(\phi) - w),
\]

where \( \eta \) is the shadow value associated with the wage constraint. The Kuhn-Tucker conditions read

\[
w + c\sigma - \sigma(w - 1) - \eta \frac{w}{l(\phi, w)} = 0,
\]

\[
\tilde{w}(\phi) - w \geq 0 \quad \text{with} \quad \eta \geq 0 \quad \text{and} \quad \eta \frac{\partial L}{\partial \eta} = 0,
\]

where we have used the fact that the labor demand elasticity is \(-\sigma\).

If the firm’s participation constraint is not binding, i.e. \( \tilde{w}(\phi) - w > 0 \) (which implies \( \eta = 0 \) by complementary slackness), we find that the wage rate is a constant mark-up over the sum of the competitive wage (i.e. the opportunity costs of working in the firm) and the marginal unionization costs (i.e. the costs of being unionized)

\[
w^u \equiv w = \frac{1}{\rho} (1 + c).
\]

We refer to this wage rate as the unconstrained wage rate denoted by \( w^u \) which is independent of the firm’s productivity because marginal unionization costs are assumed to be constant.

If the wage constraint is binding, the union sets the wage rate equal to the feasible wage rate as shown by (19), i.e. the union follows a limit wage strategy which leaves the firm with zero profits. By complementary slackness, \( \eta > 0 \) holds. Solving (18) with respect to \( \eta \) and noting \( w = \tilde{w}(\phi) \) yields

\[
\eta = l(\phi, \tilde{w}(\phi)) \left[ 1 - \sigma \frac{\tilde{w}(\phi) - 1 - c}{\tilde{w}(\phi)} \right].
\]
In appendix A.1, we show that the shadow value is positive and decreasing in $\phi$ as long as the wage constraint is binding. There exists a threshold productivity $\phi^+$ at which the wage constraint stops binding, i.e. $\eta(\phi^+) = 0$. Using (21) and (16), we can calculate this threshold productivity as

$$\phi^+ = \frac{1}{\rho^2} \left( \frac{1 + c}{P} \right) \left( \frac{\sigma F}{\nu} \right)^{\frac{1}{\sigma - 1}}. \quad (22)$$

**Lemma 1** For firms with $\phi < \phi^+$, the union sets the limit wage $\tilde{w}(\phi)$. For firms with $\phi \geq \phi^+$, the union sets the unconstrained union wage $w^u$.

Workers only choose to form a union if it pays to do so, i.e. if $\mathbb{E}(U) \geq 1$ which translates into the condition $V(w) \geq 0$. Using (12), we can calculate a minimum wage, $w^c$, which is necessary to cover the sum of opportunity costs of working and average unionization costs

$$V(w^c) = 0 \Leftrightarrow w^c = 1 + c. \quad (23)$$

The wage rate set by the union according to Lemma 1 must be at least as high as $w^c$ to support unionization. Note that $w^c$ is independent of the firm’s productivity because average unionization costs are assumed to be constant.

Comparing (20) and (23), we find that $w^u > w^c$. If the union can set the unconstrained wage, unionization is always profitable and high-productivity firms with $\phi \geq \phi^+$ are unionized. This is, however, not necessarily true for firms that pay the limit wage. Because $\tilde{w}(\phi)$ increases in productivity while $w^c$ is constant, the limit wage falls short of the union’s minimum wage for firms that are equipped with relatively low productivity levels. The firm’s profit is so low that unionization does not pay off for workers. The firm’s productivity which leads to unionization of workers at the margin – denoted by $\phi^-$ – is calculated by the intersection of $w^c$ and $\tilde{w}(\phi^-)$. Using (16) and (22), we obtain

$$\phi^- = \rho \phi^+. \quad (24)$$

**Lemma 2** For firms with $\phi < \phi^-$, there is no unionization, while the workforce of firms with $\phi \geq \phi^-$ is organized by a union.

Because the marginal firm with productivity $\phi^*$ has zero profits by definition, the union is not able to capture (at least parts of) the marginal firm’s profits to cover unionization costs. The marginal firm is, hence, not unionized, which implies that the extensive margin of the firm (whether to produce or not) is not directly affected. The resulting wage pattern $w(\phi)$ is illustrated in Figure 1 by the red line and summarized in the following Lemma.
Lemma 3  Firms with productivity

- $\phi^* \leq \phi < \phi^-$ are not unionized and pay the competitive wage $w = 1$,
- $\phi^- \leq \phi < \phi^+$ are unionized and pay the limit wage $w = \tilde{w}(\phi)$,
- $\phi \geq \phi^+$ are unionized and pay the unconstrained wage $w = w^u$.

Figure 1: Wage curve

3.3 The Entrepreneur’s Choices

Suppose that the entrepreneur has decided to enter the market, i.e. he invests $F_e$ and receives productivity $\phi$ from the draw. The entrepreneur chooses to produce if $\phi \geq \phi^*$. Profits of the marginal firm are given by $\pi(\phi^*, 1) = 0$, which is referred to as the zero-profits cutoff condition (ZPC). Using (15), we can rewrite ZPC as

$$
(1 - \rho)r(\phi^*, 1) - F_e = 0.
$$

The entrepreneur enters the market if expected profits are at least as high as the entry costs $F_e$. Due to free entry, this holds by equality in equilibrium,
which leads to the free entry condition (FE)

\[ \int_{\phi^*}^{\infty} ((1 - \rho) r(\phi, w) - F) g(\phi) d\phi = F, \]

with \( g(\phi) \) denoting the density function.

### 4 General Equilibrium

#### 4.1 Cutoff Productivity and Mass of Entrants

In analogy to Melitz (2003), we can determine the cutoff productivity and the mass of entrants in equilibrium by using the ZPC and FE conditions. Inserting (14) and (13) into the ZPC condition (25) and rearranging yields

\[ \phi^* = \frac{1}{\rho P} \left( \frac{\sigma F}{\nu} \right)^{\frac{1}{\sigma - 1}}, \]

which shows, as argued above, that unionization (costs) has no direct effect on the cutoff productivity. However, as will become clear in a moment, it changes the pricing and production behavior of all inframarginal firms and hence affects the price index \( P \). Using (27), we can derive expressions for the various wage structure thresholds, implying that \( \phi^+ = \rho^{-1}(1 + c)\phi^* \) and \( \phi^- = (1 + c)\phi^* \) (see (22) and (24)).

Because the fraction of firms sharing some common productivity level \( \phi \) is given by \( g(\phi) d\phi \), we can rewrite the CES price index (4) as

\[ P = \left[ \int_{\phi^*}^{\infty} p(\phi, w(\phi))^{-\sigma (1)} M e g(\phi) d\phi \right]^{-\frac{1}{\sigma - 1}}. \]

Determining \( P \) in equilibrium requires knowledge about the equilibrium pricing structure which follows directly from the wage structure. Inserting (28) into (27) leads then to

\[ p(\phi^*)^{-\sigma (1)} \]

\[ \left( \int_{\phi^*}^{(1+c)\phi^*} p(\phi, w_Y)^{-\sigma (1)} g(\phi) d\phi + \int_{(1+c)\phi^*}^{\rho^{-1}(1+c)\phi^*} p(\phi, \hat{w}(\phi))^{-\sigma (1)} g(\phi) d\phi \right) + \int_{\rho^{-1}(1+c)\phi^*}^{\infty} p(\phi, \hat{w})^{-\sigma (1)} g(\phi) d\phi = M e^{-1} \frac{\nu}{\sigma F}, \]

12
which gives an implicit expression of the cutoff productivity as a function of the mass of entrepreneurs willing to engage in productivity draws. An increase in \( M_e \) then leads to an increase in \( \phi^* \), intuitively, because competitive pressure increases with more entrepreneurs drawing productivities such that the cutoff productivity for the entrepreneur to start production increases.

Inserting (14) and (28) into the FE condition (26), we find

\[
M_e = \frac{\nu}{\sigma \bar{F}_e + (1 - G(\phi^*)) \bar{F}}. \tag{30}
\]

Here an increase in \( \phi^* \) implies that an entrepreneur is less likely to be able to start production, but if so receives higher operating profits. In expectations, these effects offset each other. Thus, with higher \( \phi^* \), the expected fixed costs payment decreases, which raises expected profits and the mass of entrepreneurs.

Equations (29) and (30) determine the equilibrium cutoff productivity, \( \bar{\phi}^*(c) \), and the equilibrium mass of entrants \( \bar{M}_e \). Solving explicitly for these equilibrium values (see appendix A.2 for the derivation) gives

\[
\bar{\phi}^*(c) = \phi_{\text{min}} \left[ (\Gamma_1(c) + \Gamma_2(c)) \frac{\bar{F}}{\bar{F}_e} \right]^{\frac{1}{k}}, \tag{31}
\]

\[
\Gamma_1(c) \equiv (w^e(c))^{-k} - \frac{k}{k - \xi} (w^e(c))^{-(k - \xi)},
\]

\[
\Gamma_2(c) \equiv \frac{\xi}{k - \xi} (1 + (w^u(c))^{-k}), \tag{32}
\]

with \( \xi \equiv \sigma - 1 \) and \( k > \xi \). Inserting (30) into \( M = (1 - G(\phi^*))M_e \) determines the equilibrium mass of active firms (and the mass of available varieties)

\[
\bar{M}(c) = \frac{\nu}{\bar{c} \left(1 - G(\bar{\phi}^*(c))\right)^{-1} \bar{F}_e + \bar{F}}. \tag{33}
\]

The equilibrium price index \( \bar{P}(c) \) follows from rearranging (27).

**4.2 Welfare**

Welfare is measured by the indirect utility

\[
\bar{U}(\bar{X}(c), \bar{Y}(c)) = \nu ln(\bar{X}(c)) + \bar{Y}(c) = \nu ln \left( \frac{\nu}{\bar{P}(c)} \right) + \bar{I}(c) - \nu, \tag{34}
\]
where we have used (3) and (2). Income is defined as $I = Y + PX$. Because production in the $Y$-sector equals $L_Y = Y + MF + M_e F_e + \int_{\phi^-}^\infty c(\phi) M_e g(\phi) d\phi$, income can be rewritten as

$$I = L_Y - MF - M_e F_e - \int_{\phi^-}^\infty c(\phi) M_e g(\phi) d\phi + PX,$$

where $\int_{\phi^-}^\infty c(\phi) M_e g(\phi) d\phi$ denotes aggregate unionization costs. Using the FE condition (which implies that aggregate profits minus market entry costs are zero) and $L = L_Y + L_X = L_Y + \int_{\phi^*}^\infty l(\phi) M_e g(\phi) d\phi$, equilibrium income reads:

$$\bar{I}(c) = L + \int_{\phi^*}^\infty w(\phi, c) l(\phi, c) M_e g(\phi) d\phi - \int_{\phi^*}^\infty l(\phi, c) M_e g(\phi) d\phi - \int_{\phi^-}^\infty c(\phi, c) M_e g(\phi) d\phi,$$

(35)

with $\bar{V}_{\text{agr}}(c)$ representing the aggregate net utility of unionization (see (12)). If unionization costs go to infinity, $\bar{V}_{\text{agr}}$ is zero and income equals (the value of) labor endowment.

## 5 The Impact of Unionization Costs

### 5.1 Analytical Solution

Suppose that unionization costs $c$ increase due to a ‘deunionization’ policy that is implemented by the government, for example by making it harder to form a union. The effect on the productivity structure is then summarized in proposition 1.

**Proposition 1** Equilibrium cutoff productivity decreases in $c$ for low unionization costs, reaches a minimum at $c_{\text{crit}}$ and increases from there on.

**Proof 1** Differentiating $\tilde{\phi}^*$ with respect to $c$ (see appendix A.3) yields

$$\frac{\partial \tilde{\phi}^*}{\partial c} = \frac{1}{k \Gamma_1 + \Gamma_2} \left[ \frac{\partial \Gamma_1(w^c)}{\partial c} > 0 + \frac{\partial \Gamma_2(w^u)}{\partial c} < 0 \right].$$

(36)
Using (32), we obtain

\[
\frac{\partial \hat{\phi}^*}{\partial c} \left\{ \begin{array}{l} < \\ \geq \end{array} \right\} 0 \iff c \left\{ \begin{array}{l} < \\ \geq \end{array} \right\} c_{\text{crit}},
\]

\[
c_{\text{crit}} \equiv c = \left(1 + \frac{\xi \rho^k}{k - \xi}\right)^{\frac{1}{k}} - 1 > 0.
\]

There are two countervailing effects. First, an increase in \(c\) raises the minimum wage \(w^c\) for unionization. This implies that a higher share of firms is not unionized such that more firms pay the competitive wage (instead of the union wage). Those firms can reduce their prices such that competition becomes more fierce and the least productive firms are driven out of the market; \(\hat{\phi}^*\) increases. Second, an increase in \(c\) raises the unconstrained wage. (High-productivity) Firms that face this wage increase set higher prices such that competition becomes less intense and more entrepreneurs with lower productivities are able to start production; \(\hat{\phi}^*\) decreases.

If unionization costs are relatively low at the outset, a large share of firms is unionized. This implies that quantitatively more firms are confronted with the increase in the unconstrained wage (second effect) than with the wage decrease (first effect). Consequently, \(\hat{\phi}^*\) decreases in \(c\). If, however, unionization costs are initially relatively high, there are only few unionized firms such that the reverse argumentation holds and \(\hat{\phi}^*\) increases in \(c\).

We consider two polar cases. With \(c \to \infty\), we have \(\lim_{c \to \infty} \hat{\phi}^- \to \infty\) such that no firm is unionized. With \(\hat{\phi}^-(c = 0) = \phi^*\), all firms are unionized.

**Proposition 2** Cutoff productivity in an industry with no unionization \((c \to \infty)\) is higher than in an industry with full unionization \((c = 0)\).

**Proof 2** If \(c \to \infty\), all firms pay the competitive wage. If \(c = 0\), \(w^c(c = 0) = 1\) and \(w^u(c = 0) = \rho^{-1}\) hold. Using (31), we obtain

\[
\frac{\hat{\phi}^*_{c \to \infty}}{\hat{\phi}^*_{c = 0}} = \frac{\rho^{-1} \phi_{\text{min}} \rho \left[ \xi F/(F_e(k - \xi)) \right]^{1/k}}{\phi_{\text{min}} \rho \left[ \xi F/F_e(k - \xi) \right]^{1/k}} = \rho^{-1} > 1.
\]

An increase in unionization costs also affects the mass of entrants and the mass of available varieties.

**Proposition 3** (i) The equilibrium mass of entrants is U-shaped in unionization costs. (ii) The equilibrium mass of available varieties is hump-shaped in unionization costs.
**Proof 3** Using (30) and (33), we find
\[ \frac{\partial \bar{M}_e}{\partial c} = \frac{\partial \bar{M}_e}{\partial \bar{\phi}^*} \frac{\partial \bar{\phi}^*}{\partial c}, \quad \frac{\partial \bar{M}}{\partial c} = \frac{\partial \bar{M}}{\partial \bar{\phi}^*} \frac{\partial \bar{\phi}^*}{\partial c}. \]

Together with proposition 1, this proves the proposition.

A change in \( c \) affects the mass of entrants and the mass of available varieties indirectly through the change in the cutoff productivity. The direct effect of \( c \) on \( \bar{\phi}^* \) is U-shaped, as argued above. An increase in cutoff productivity raises \( M_e \) because expected profits increase. The mass of firms, however, decreases because of a more intense firm-selection.

A change in unionization costs also affects welfare.

**Proposition 4** Equilibrium consumption \( \bar{Y} \) declines in \( c \). Equilibrium consumption \( \bar{X} \) is U-shaped in \( c \). If \( c \leq c_{\text{crit}} \), \( \bar{X} \) declines in \( c \), which implies that \( \bar{U} \) decreases. If, however, \( c > c_{\text{crit}} \), \( \bar{X} \) increases in \( c \), making the overall effect on \( \bar{U} \) ambiguous.

**Proof 4** Differentiating (34) with respect to \( c \) yields
\[ \frac{\partial W}{\partial c} = \nu \frac{\partial \bar{X}}{\partial c} + \frac{\partial \bar{Y}}{\partial c}. \]

From (5), (27) and (2), we obtain
\[ \frac{\partial \bar{X}}{\partial c} = \frac{\partial \bar{X}}{\partial \bar{P}} \frac{\partial \bar{P}}{\partial \bar{\phi}^*} \frac{\partial \bar{\phi}^*}{\partial c}, \quad \frac{\partial \bar{Y}}{\partial c} = \frac{\partial \bar{I}}{\partial c}. \]

As shown in appendix A.4, \( \partial \bar{I}/\partial c < 0 \) holds. Together with proposition 1, this proves the proposition.

Consumption in the \( X \)-sector varies with unionization costs because they affect the cutoff productivity which in turn influences the price index. Intuitively, an increase in the cutoff productivity has two effects on \( \bar{P} \). On the one hand, operating firms are on average more productive such that the price index decreases. On the other hand, the mass of available varieties decreases which raises the price index. Analytically, the former effect dominates such that an increase in the cutoff productivity reduces \( \bar{P} \) and hence raises \( \bar{X} \).

Consequently, the relation between \( \bar{X} \) and \( c \) is U-shaped.

\[ ^{10} \text{This is the standard welfare effect of changes in productivity. Economies that are more productive produce cheaper, the price index decreases and hence consumption of the differentiated good increases.} \]
Consumption in the $Y$-sector is affected by changes in unionization costs because they have an impact on equilibrium income. As $c$ increases, the (net) utility of each active union and the fraction of unionized firms decrease. Both imply a reduction of $\bar{V}^\text{aggr}$ and thus $\bar{I}$.

If unionization costs are relatively high, i.e. $c > c^{\text{crit}}$, there are two countervailing effects on welfare. An increase in $c$ causes $\bar{X}$ to rise on the one hand, while consumption in the $Y$-sector declines on the other. The net effect on welfare is ambiguous. If unionization costs are relatively low, i.e. $c < c^{\text{crit}}$, consumption in the $X$-sector declines in $c$. Because income also decreases, welfare is reduced through an increase in unionization costs.

5.2 Numerical Solution

5.2.1 Calibration

We solve our model numerically to gain insights into the quantitative importance of the aforementioned effects. Additionally, we analyze wage inequality effects of unionization. Due to the complexity of the model, this was not possible for the analytical solution. Learning about the inequality effects of unionization is important, because it often lies at the heart of policy debates.

We follow a frequently used approach and calibrate our model with parameter values drawn from the literature. As such, our results are comparable to other studies that use the same parameter values. Specifically, we rely on the calibration by Bernard et al. (2007) and set $\sigma = 3.8$, $\delta = 0.025$ and $F_e = 2.11$. Additionally, we take the results of the structural estimations by Balistreri et al. (2011) into account and set $k = 4.6$ as well as $F = 0.25$ (which is the average value of estimated fixed costs in the US and Europe). We normalize labor endowment to $L \equiv 1.5$ and assume the relative size of the $X$-sector to be $\nu \equiv 0.5$ as also done by Helpman and Itskhoki (2010). The former ensures that $L_X < L$ such that there are labor units left to produce $Y$. The latter implies the $X$-sector is relatively small compared to the $Y$-sector.

5.2.2 Productivity Distribution and Welfare

As shown by Figure 2, an increase in $c$ leads to the U-shaped relationship between $c$ and $\bar{\phi}^*$ respectively $c$ and $\bar{X}$ (see propositions 1 and 4). In addition,
we find that \( c^{crit} = 0.12 \). This implies that the equilibrium cutoff productivity and equilibrium consumption in the \( X \)-sector increase in \( c \) if \( c > 0.12 \).\(^{12}\)

In Figure 3, the effects of unionization costs on equilibrium consumption of good \( Y \) and on welfare are illustrated. \( \bar{Y} \) decreases in \( c \) which is driven by the fact that income monotonously declines. This has, c.p., a welfare-reducing effect which turns out to be strong for low values of unionization costs. With respect to welfare, we find that \( \bar{U} \) decreases for low values of \( c \) because both \( \bar{Y} \) and \( \bar{X} \) decline. If \( c > 0.18 \), welfare increases in \( c \) because the reduction of \( \bar{Y} \) is then overcompensated by the increase in \( \bar{X} \).

\(^{12}\)Recall that \( c \) (= average unionization costs) is measured in units of the opportunity costs of working. So, in our simulation, the sign of the effect of unionization on productivity switches if average unionization costs exceed 12\% of these opportunity costs.
5.2.3 Wage Inequality

We choose the Gini coefficient as the measure of wage inequality (see Egger and Kreickemeier, 2012, Helpman et al., 2010 for a similar approach). Determining the Gini coefficient requires deriving the Lorenz curve. This depicts the wage income of the bottom \( \theta \) percent of employed workers in the \( X \)-sector relative to overall wage income in this sector.

The cumulated wage sum in firms in which productivity is smaller than \( \hat{\phi} \) is given by

\[
W_X(\hat{\phi}) = \int_{\hat{\phi}}^{\phi_*} w(\phi)l(\phi)M_c g(\phi) d\phi, \tag{37}
\]

Denoting cumulated employment in these firms by \( L_X(\hat{\phi}) = \int_{\hat{\phi}}^{\phi_*} l(\phi)M_c g(\phi) d\phi \), we get the fraction of employment as \( \frac{L_X(\hat{\phi})}{L_X(\infty)} \). Hence, the lower \( \theta \) percent (the \( \theta \) percentile) of employment is

\[
\theta = \frac{L_X(\hat{\phi})}{L_X(\infty)}. \tag{38}
\]

Equation (38) implicitly results in a function \( H(\cdot) \) which determines the threshold productivity \( \hat{\phi} \) as a function of the percentile \( \theta \) which then is \( \hat{\phi} = H(\theta) \). Using this, the relative wage sum of the lower \( \theta \) percent of employment is

\[
\frac{W_X(H(\theta))}{W_X(H(1))} = \frac{\int_{\hat{\phi}}^{H(\theta)} w(\phi)l(\phi)M_c g(\phi) d\phi}{\int_{\hat{\phi}}^{H(1)} w(\phi)l(\phi)M_c g(\phi) d\phi}. \tag{39}
\]

This gives the Lorenz curve, which can be used to calculate the Gini coefficient defined as

\[
Gini = 1 - 2 \int_0^1 \frac{W_X(H(\theta))}{W_X(H(1))} d\theta \tag{40}
\]

with \( Gini \in [0, 1] \). The higher the Gini coefficient the more unequal the wage distribution.

Figure 4 shows the numerical computation of the Gini coefficient for different values of unionization costs. There is a hump-shaped relation between the Gini coefficient and \( c \). Starting at a point where all firms are unionized (\( c = 0 \), an increase in unionization costs implies that high-productivity firms have to pay higher wages (the unconstrained wage increases), while some low-productivity firms are not unionized and can reduce their wage payments. The (limit) wage paid by intermediate-productivity firms is not affected. Consequently, wage inequality rises. With further increases in unionization costs, more and more firms are not unionized and pay the competitive wage, while only few firms pay the limit wage and even fewer the
(relatively high) unconstrained wage. This makes the wage distribution more equal. If unionization costs go to infinity, all firms pay the competitive wage and wage inequality vanishes.

### 6 Robustness

Our model is characterized by two crucial assumptions: monopoly unions and linear unionization costs. In this section, we relax these assumptions by introducing Nash bargaining at the firm-level and by considering alternative unionization cost functions. Our goal is to show that these modifications lead to qualitatively similar wage paths (see Lemma 3) such that we can argue that the qualitative nature of the equilibrium structure remains intact.

#### 6.1 Nash Bargaining

Suppose that the union and the firm bargain over the wage, while the firm sets employment. The optimization problem is

\[
\max_w N = (V(l, w) - \hat{V})^\gamma (\pi(\phi, l, w) - \hat{\pi})^{1-\gamma},
\]

s.t. \( l = l(\phi, w) \),

where \( N \) denotes the Nash product consisting of the union’s bargaining gain, \( V(l, w) - \hat{V} \), and of the firm’s bargaining gain, \( \pi(\phi, l, w) - \hat{\pi} \), weighted by the parties bargaining power \( \gamma \) and \( 1 - \gamma \), \( 0 < \gamma \leq 1 \). If the bargaining fails,
the union’s and the firm’s payoff are \( \hat{V} = 0 \) and \( \hat{\pi} = 0 \), respectively.\(^{13}\)

The first-order condition of that problem reads

\[
E \equiv \frac{1}{\rho} (1 + c)(w^b)^{-1} - 1 - \frac{1 - \gamma}{(\sigma - 1)\gamma} \frac{V(l(\phi, w^b))}{\pi(l(\phi, w^b))} = 0, \tag{41}
\]

which implicitly pins down the bargained wage \( w^b \). The second-order condition, \( \partial E/\partial w \equiv E_w < 0 \), is fulfilled. \( \gamma = 1 \) resembles the monopoly union case. In contrast to this case, we do not have to explicitly consider the restriction that the firm’s profit is non-negative, because this is ensured by the Nash bargaining approach. Hence, the branch of the wage path that have been labeled ‘unconstrained’ and ‘limiting’ in the previous sections are merged into one wage path. However, we do not find then an explicit solution for \( w^b \).

Totally differentiating (41) allows us to characterize the form of the wage path

\[
\frac{dw^b}{d\phi} = -\frac{E_{\phi}}{E_w} > 0, \tag{42}
\]

\[
E_{\phi} \equiv \frac{\partial E}{\partial \phi} = -\frac{1 - \gamma V}{\gamma \phi} \left( 1 - \frac{(1 - \rho)r(\phi, w)}{(1 - \rho)r(\phi, w) - F} \right) > 0. \tag{43}
\]

This shows that the bargained wage is an increasing function of the firms’ productivities. The least productive firms are not unionized and pay the competitive wage while firms with sufficiently large productivity levels are unionized and pay the bargained wage. As a result, the wage pattern is qualitatively similar to our benchmark model.\(^{14}\)

6.2 Unionization Costs
6.2.1 A General Approach

Assume that unionization costs are given by \( C = C(l) \) and that marginal, \( C'(l) \), as well as average unionization costs, \( C(l)/l \), are positive. The union’s

\(^{13}\)Since payoffs are made at the last stage of the game, unionization costs are only paid if the bargaining is successful as such \( \hat{V} = 0 \). Because we assume that firm’s fixed costs are reversible, the firm’s outside option is zero which implies \( \hat{\pi} = 0 \).

\(^{14}\)In some studies, see for instance Eckel and Egger (2009), it is assumed that the firm’s payoff in case of the bargaining’s fail is \( \hat{\pi} = -F \). The bargained wage then does not depend on the firms’ productivities. Identical wages among heterogeneous firms are, however, in contrast to the empirical evidence. Moreover, it would imply that either all firms are unionized or no firm is unionized, which is also rather the exception than the rule.
optimization problem for setting the wage is given by
\[
\max_w V = l(w - 1) - C(l)
\]
\[\text{s.t.} \quad l = l(\phi, w) \quad \text{and} \quad w \leq \hat{w}(\phi).\]

The Lagrange function and the Kuhn-Tucker conditions read
\[
L = l(\phi, w)(w - 1) - C(l(\phi, w)) + \eta(\hat{w}(\phi) - w),
\]
\[w + \sigma C'(l(\phi, w)) - \sigma(w - 1) - \eta \frac{w}{l(\phi, w)} = 0,\]
\[\hat{w}(\phi) - w \geq 0 \quad \text{with} \quad \eta \geq 0 \quad \text{and} \quad \eta \frac{\partial L}{\partial \eta} = 0.\]

We assume that the second-order conditions are fulfilled.

Suppose that the wage constraint is non-binding such that \(\eta = 0\). Using (45), we find that the unconstrained wage, \(w^u\), is given by the solution of:
\[
D \equiv -w^u + 1 + \frac{1}{\rho} \left[1 + C'(l(\phi, w^u))\right] = 0.
\]

This implies that we cannot determine the unconstrained wage explicitly. However, we are able to calculate the path of the unconstrained wage in the \((w^u, \phi)\)-space. Totally differentiating (47) yields:
\[
\frac{dw^u}{d\phi} = -\frac{D_\phi}{D_{w^u}},
\]
where subscripts denote partial derivatives. Due to the second-order condition, \(D_{w^u} < 0\) holds. Moreover, we get \(D_\phi = \rho^{-1}C''(l(\phi, w^u))l_\phi\) with \(l_\phi > 0\).

**Lemma 4** If marginal unionization costs are increasing, \(C''(l(\phi, w^u)) > 0\), the unconstrained wage increases in \(\phi\) and vice versa.

If the wage constraint is binding, the union follows the limit wage strategy and sets \(w = \hat{w}(\phi)\). From (45), we obtain
\[
\eta(\phi) = l(\phi, \hat{w}(\phi)) \left[1 - \sigma \frac{\hat{w}(\phi) - 1 - C'(l(\phi, \hat{w}(\phi)))}{\hat{w}(\phi)}\right].
\]

Using \(l(\phi, \hat{w}(\phi)) = \frac{(\sigma - 1)F}{\hat{w}(\phi)}\), this can be rewritten as:
\[
\eta(\phi) = (\sigma - 1)F \left[\frac{1 + C'(l(\phi, \hat{w}(\phi)))}{\hat{w}(\phi)^2} - \frac{\sigma - 1}{\hat{w}(\phi)}\right].
\]
Differentiating (50) with respect to $\phi$ yields:

\[
\frac{d\eta}{d\phi} = \left(\sigma - 1\right) \frac{F}{\bar{w}(\phi)} \begin{cases} \left[1 - 2 \frac{w^u(\phi)}{\bar{w}(\phi)}\right] - \sigma F \bar{w}(\phi)^{-2} C''(l(\phi, \bar{w}(\phi))) & \text{if } \bar{w}(\phi) > 0 \\ \left[1 - 2 \frac{w^u(\phi)}{\bar{w}(\phi)}\right] - \sigma F \bar{w}(\phi)^{-2} C''(l(\phi, \bar{w}(\phi))) & \text{if } \bar{w}(\phi) < 0\end{cases}.
\]

We can also use (49) to implicitly compute the threshold productivity $\phi^+$ at which $\eta(\phi^+) = 0$ and obtain

\[
\bar{w}(\phi^+) - \frac{1}{\rho} \left[1 + C''(l(\phi^+, \bar{w}(\phi^+)))\right] = 0.
\]

The equilibrium wage path, which we denote by $w^{\text{union}}(\phi)$ (this comprises of the limit wage and the unconstrained union wage) is characterized by the following lemma.

**Lemma 5**  
- If marginal unionization costs are decreasing, the unconstrained union wage $w^u(\phi)$ decreases. For $\bar{w}(1) < w^u(1)$ (which we assume to hold) there exists a productivity $\phi^+$ for which limit wage and unconstrained wage path intersect. For firms with $\phi < \phi^+$, the union sets the limit wage. For firms with $\phi \geq \phi^+$, the union sets the unconstrained wage.

- If marginal unionization costs are increasing, the shadow value declines in $\phi$ such that there exists a threshold productivity $\phi^+$ defined by $\eta(\phi^+) = 0$. For firms with $\phi < \phi^+$, the union sets the limit wage. For firms with $\phi \geq \phi^+$, the union sets the unconstrained wage.

Workers decision to form a union is also affected by unionization costs. Unionization will occur if

\[
V \geq 0 \iff w^{\text{union}}(\phi) - \frac{1}{1 + C'(l(\phi, w^{\text{union}}(\phi)))} \geq 0.
\]

The change of union utility in $\phi$ is given by

\[
\frac{dV}{d\phi} = \frac{\partial w^{\text{union}}}{\partial \phi} - \left(1 - \frac{1}{\frac{C'(l(\phi, w^{\text{union}}(\phi)))}{l(\phi, w^{\text{union}}(\phi))}}\right) (\epsilon_{C_l} - 1) \phi^{-1} \epsilon_{l\phi},
\]

where $\epsilon_{C_l}$ denotes the elasticity of the union cost function with respect to employment and $\epsilon_{l\phi}$ denotes the (partial) elasticity of $l$ with respect to changes
in $\phi$. Equation (53) depicts the two countervailing forces that affect union’s utility as $\phi$ changes. This is comprised of the direct wage effect and the change in average costs, which are a function of the form of the cost function and the (direct and indirect) change of employment. The general effect remains ambiguous. The wage as well as average costs may decrease or increase with productivity $\phi$. Identifying those firm productivities for which unionization occurs is, without any additional structure, impossible. Note that in the benchmark case with linear unionization costs, only the direct wage effect of productivity changes remains relevant and greatly simplifies calculations.

### 6.2.2 Three Examples

Because we cannot explicitly determine the wage path, we provide three examples and solve for the wage path numerically. Assume at first that unionization costs are given by:

$$C = cl^{\alpha_i},$$

with $i = 1, 2$. For the case of increasing marginal and average unionizations costs, $\alpha_1 = 1.5$ holds. For the case of decreasing marginal and average unionization costs, we set $\alpha_2 = 0.9$. The simulation results are illustrated in Figure 5, where the red line depicts the wage path.

![Figure 5: Decreasing/Increasing marginal and average union costs](image)

This shows that the wage path is qualitatively similar compared to our benchmark model. Low-productivity firms are not unionized, intermediate-productivity firms are unionized and pay the limit wage while high-productivity firms pay the unconstrained wage. Note, however, that $w^u$ (slightly) declines in the case of decreasing marginal unionization costs.
Let us now turn to the case where organizing the firm’s workforce requires fixed costs $K$

$$C = cl + K,$$

with $K = 0.5$. Marginal unionization costs are constant as in our benchmark model but average unionization costs decline. As illustrated in Figure 6, this implies that a lower fraction of firms is unionized compared to our benchmark model, but that the remaining wage path is not affected.

Figure 6: Fixed unionization costs

7 Conclusion

In this paper, we analyze how an increase in unionization costs affects the distribution of firms within an industry, sectoral wage inequality and welfare. Our theoretical framework combines the Melitz (2003) model with heterogeneous firms, monopolistic competition and free market entry/exit with labor unions that set wages at the firm-level. Because organizing the firms’ workforces is associated with unionization costs, unionization is an endogenous outcome. Similar to Kuhn (1988), we find that only sufficiently productive firms are unionized while low-productivity firms pay the competitive wage.

As our main results, we find that the relation between the cutoff productivity and unionization costs is U-shaped and that the relation between sectoral wage inequality and unionization costs is hump-shaped. On the one hand, an increase in unionization costs is carried over from unions to firms in the course of setting the wage. Hence, wages and prices increase. On the other hand, firms are freed from unionization which enables them to pay lower wages and decrease prices.
If unionization costs are low at the outset, i.e. a large number of firms is unionized, the price increasing effect of higher unionization costs dominates. Competition becomes less intense and more low-productivity firms can survive in the market, which encourages entry for lower productive entrepreneurs. This implies also that wage inequality rises. With further increases in unionization costs, the mass of firms paying the union wage decreases while there are more firms for which the workforce is not unionized anymore and that pay the competitive wage. This reduces wage inequality as well as wages and prices on average. Competition becomes more severe and less low-productivity firms are active in the market.

Welfare can also decrease in unionization costs. This is caused by two effects. First, income and thus the consumption of the homogenous good monotonously decrease in unionization costs. Second, if unionization costs are low, the reduced cutoff productivity implies that firms are on average less productive which raises the price index and thus reduces consumption of the differentiated good. Only if unionization costs are sufficiently high a welfare-enhancing effect of lower unionization exists due to the implied increase in the cutoff productivity and thus an increase in consumption of the differentiated good. From a policy perspective, these results should be taken into account when deciding about a change in the legal barriers to unionization. Making the labor market more competitive by restricting unionization, as frequently observable in developed countries, can but not necessarily has to be an improvement in economic performance in terms of sectoral wage inequality and welfare.

A Appendix

A.1 Shadow Value of the Wage Constraint

Observing (21), we find that $\eta > 0$ if and only if

$$1 - \sigma \frac{\tilde{w}(\phi) - 1 - c}{\tilde{w}(\phi)} > 0$$

$$\Leftrightarrow \tilde{w}(\phi) < \frac{\sigma}{\sigma - 1}(1 + c) = \frac{1}{\rho(1 + c)}.$$

Given the unconstrained wage (20), this condition holds in case of $\tilde{w}(\phi) < w^u$ and thus as long as the wage constraint is binding.

To determine the slope of the shadow value in the $(\phi, \eta)$-space, we first
use $\pi(\phi, \tilde{w}(\phi)) = 0$ and (15) to compute

$$l(\phi, \tilde{w}(\phi)) = \frac{(\sigma - 1)F}{\tilde{w}(\phi)}. \quad (A-1)$$

Inserting (A-1) into (21), we can rewrite the shadow value as

$$\eta(\phi) = (\sigma - 1)F \left[ \frac{(1 + c)\sigma}{\tilde{w}(\phi)^2} - \frac{\sigma - 1}{\tilde{w}(\phi)} \right]. \quad (A-2)$$

Differentiating (A-2) with respect to $\phi$ leads to

$$\frac{\partial \eta}{\partial \phi} = \left( \frac{\sigma - 1}{\tilde{w}(\phi)} \right)^2 F \frac{\partial \tilde{w}(\phi)}{\partial \phi} \begin{cases} 1 - 2 \frac{w^u}{\tilde{w}(\phi)} > 0 \\ < 0 \end{cases}$$

if $\tilde{w}(\phi) < w^u$, i.e. in case the wage constraint is binding.

### A.2 Equilibrium Cutoff Productivity

Inserting (30) into (29) leads to

$$A(\phi^*) = \frac{F_c}{F} + (1 - G(\phi^*)), \quad (A-3)$$

where $A$ is defined as the LHS of (29). This can be rewritten as

$$A = (\phi^*)^{-\xi} \left[ \int_{\phi^-}^{\phi^*} \phi g(\phi)d\phi + \int_{\phi^-}^{\phi^*} \left( \frac{\tilde{w}(\phi)}{\phi} \right)^{-\xi} g(\phi)d\phi + (w^u)^{-\xi} \int_{\phi^-}^{\phi^*} \phi g(\phi)d\phi \right],$$

with $\xi = \sigma - 1$. Using (16) and (27), we obtain: $\tilde{w}/\phi = 1/\phi^*$. This leads to

$$A = (\phi^*)^{-\xi} \left[ \int_{\phi^-}^{\phi^*} \phi g(\phi)d\phi + (\phi^*)^{\xi} \int_{\phi^-}^{\phi^*} g(\phi)d\phi + (w^u)^{-\xi} \int_{\phi^-}^{\phi^*} \phi g(\phi)d\phi \right]. \quad (A-4)$$

Noting $\phi^+ = w^u \phi^*$ as well as $\phi^- = w^c \phi^*$, we can solve the integrals and obtain

$$\int_{\phi^-}^{\phi^*} \phi g(\phi)d\phi = \frac{k}{k - \xi} \phi_{min}(\phi^*)^{-(k-\xi)} \left( 1 - (w^c)^{-(k-\xi)} \right),$$

$$\int_{\phi^-}^{\phi^*} g(\phi)d\phi = \phi_{min}(\phi^*)^{-k} \left[ (w^c)^{-k} - (w^u)^{-k} \right],$$

$$\int_{\phi^-}^{\phi^*} \phi g(\phi)d\phi = \frac{k}{k - \xi} \phi_{min}(w^u \phi^*)^{-(k-\xi)}.$$
Inserting these expressions into (A-4), we find after some rearrangements

\[
A = k \phi_{\text{min}}(\phi)^{-k} \left[ (w^c)^{-k} - \frac{k}{k - \xi} (w^c)^{-k} - \frac{\xi}{k - \xi}(w^u)^{-k} + \frac{k}{k - \xi} \right].
\]

(A-5)

Combining (A-5) and (A-3) leads to the equilibrium cutoff productivity as given by (31).

### A.3 Proof of Proposition 1

Differentiating \( \tilde{\phi}^* \) with respect to \( c \) yields

\[
\frac{\partial \tilde{\phi}^*}{\partial c} = \frac{1}{k} \frac{\tilde{\phi}^*}{\Gamma_1 + \Gamma_2} \left[ \frac{\partial \Gamma_1}{\partial c} + \frac{\partial \Gamma_2}{\partial c} \right].
\]

(A-6)

Using (32), we obtain

\[
\frac{\partial \Gamma_1}{\partial c} = k \left[ (w^c)^{k - \xi} (w^c)^{-k - 1} > 0, \right.
\]

\[
\frac{\partial \Gamma_2}{\partial c} = -\frac{k \xi \rho^{-1}}{k - \xi} (w^u)^{-k - 1} < 0.
\]

Inserting these expressions into (A-6) shows that

\[
\text{sign} \left[ \frac{\partial \tilde{\phi}^*}{\partial c} \right] = \text{sign} \left[ (1 + c)^{k - \xi} - \left( 1 + \frac{\xi \rho^k}{k - \xi} \right) \right].
\]

This pins down the critical level of unionization costs, \( c^{\text{crit}} \), and thus proves proposition 1.

### A.4 Proof of Proposition 4

Equilibrium income reads \( \bar{I} = L + \bar{V}^{\text{agr}} \) (see (35)). Rewriting the aggregate net utility of unions yields

\[
\bar{V}^{\text{agr}} = M_e \left[ \int_{\bar{\phi}^- (c)}^{\bar{\phi}^+ (c)} \tilde{V}(\phi, c) g(\phi) d\phi + \int_{\phi^+ (c)}^{\infty} V^u(\phi, c) g(\phi) d\phi \right],
\]

(A-7)

where \( \tilde{V}(\phi, c) = \bar{V}(\phi, c)(\bar{w}(\phi, c) - 1) - \bar{c} \bar{l}(\phi, c) \) is the net utility of unions paying the limit wage and \( V^u(\phi, c) = l^u(\phi, c)(w^u(c) - 1) - c l^u(\phi, c) \) equals the net utility of unions paying the unconstrained wage.
Differentiating $\phi^- = (1 + c)\phi^*(c)$ with respect to $c$ leads to

$$\frac{\partial \phi^-}{\partial c} = \phi^* + (1 + c)\frac{\partial \phi^*}{\partial c}$$

$$= \phi^* + (1 + c)(1 + c)^{-k-1} \frac{\phi^*}{\Gamma_1 + \Gamma_2} \left[ (1 + c)^{\xi} - 1 - \frac{k\rho^k}{k - \xi} \right]$$

(A-8)

$$= \phi^* \left[ 1 + \frac{1}{\Gamma_1 + \Gamma_2} \left( (1 + c)^{-(k-\xi)} - (1 + c)^{-k} - \frac{k\rho^k(1 + c)^{-k}}{k - \xi} \right) \right].$$

Inserting (32), we can show that the term in the squared bracket is positive, which implies $\partial \phi^- / \partial c > 0$. An increase in $c$ reduces the fraction of firms that are unionized. Note that $\phi^+$ increases, too, because $\phi^+ = \rho^{-1}\phi^-$. Using (16), (27) and (3), we can show that $\tilde{w}(\phi, c) = \phi/\phi^*(c)$ and $\tilde{l}(\phi, c) = (\sigma - 1)F/\tilde{w}(\phi, c)$. This implies that $\tilde{V}(\phi, c) = (\sigma - 1)F(1 - \phi^-/(c)\phi^-).$ Analogously, we can use (20) (27) and (3) to calculate $l^u(\phi, c) = (\sigma - 1)F\rho^\sigma \phi^* - \phi^*(-(\sigma - 1)).$ This leads to $V^u(\phi, c) = F\phi^\sigma - \rho^\sigma \phi^*(c)^{-1}(1 + c)$. Differentiating $V$ and $V^u$ with respect to $c$ yields

$$\frac{\partial V}{\partial c} = -(\sigma - 1)F \frac{1}{\phi} \frac{\partial \phi^-}{\partial c} > 0,$$

$$\frac{\partial V^u}{\partial c} = -(\sigma - 1)F\rho^\sigma \phi^* \frac{1}{\phi^*(c)} \frac{\partial \phi^-}{\partial c} > 0.$$

An increase in $c$ reduces the utility of unions. Together with the increase in $\phi^-$, this proves that $\tilde{V}_{agr}$ declines in $c$ (see (A-7)), as postulated in proposition 4.15

15Because the relation between $M_e$ and $c$ is U-shaped, there is a countervailing effect on $V_{agr}$ if $c > c^{crit}$. This effect, however, does not compensate for the reduction in $V$ and $V^u$. 

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