The Redistribution of Trade Gains and the Equity-Efficiency Trade-Off

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Abstract

The contribution of this paper is to derive an optimal redistribution scheme for trade gains in the case of a government’s objective function that explicitly accounts for the equity-efficiency trade-off. The government pays unemployment benefits (UB) either financed by a wage tax, a payroll tax or a profit tax paid by exporters only. Using a Melitz-type framework with unionized labor markets and heterogeneous workers we show that there is a clear-cut ranking of the redistribution schemes in terms of welfare level: 1. UB financed by a profit tax paid by exporters, 2. UB financed by a wage tax, 3. UB financed by a payroll tax.

JEL-Classification: F1, F16, H2

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1 Introduction

Recent empirical studies, e.g. by Amiti and Davis (2012) and Goldberg and Pavcnik (2007), confirm that trade liberalization is attended by rising income inequality. Scheve and Slaughter (2007) as well as the OECD (2008) argue that these distributional issues are critical because they raise resistance to free trade and so policy makers might be forced to increase the degree of protectionism. To ensure support for a policy of international integration, the government should pursue the goal of equality and thus redistribute income towards low-income workers. Designing such a redistribution scheme (henceforth RS) is an issue which is frequently discussed in the literature. Most of the prevailing studies, however, point out that income redistribution comes at a price: there is a decline in aggregate income per capita or more generally a loss of efficiency (see, for instance, Davidson/Matusz, 2006; de Pinto, 2012; Harrison et al., 2003).

Despite the fact that the trade-off between equity and efficiency is well-known, introducing a distributional argument into the government’s objective function is an exception rather than the rule. It is common to assume that the government’s objective function is identical with the (traditional) welfare function which includes aggregate income per capita exclusively. There is, however, a large body of literature on income inequality aversion which raises concerns about the completeness of the traditional welfare measurement. For instance, Thurow (1971) argues that the income distribution is an explicit argument in the individuals’ utility functions and shows theoretically that income redistribution can even achieve a Pareto improvement in this case. Fehr and Schmidt (1999) use an utility function which also accounts for inequality aversion and find strong experimental evidence for its existence. In more recent empirical studies, D’Ambrosio and Frick (2012) as well as Engelmann and Strobel (2004) confirm the importance of the inequality aversion criteria. Thus, one might argue that an objective function which disregards the equity-efficiency trade-off implies distorted results; the RS cannot be optimal.

The contribution of this paper is to derive an optimal RS where the equity-efficiency trade-off is explicitly entered into the government’s objective function. In line with the literature, we also assume that the government’s objective function is identical with the welfare function. In our model, however, welfare is measured by a combination of aggregate income (positive argument) and income inequality (negative argument). We restrict the government’s expenditure side to unemployment benefits (henceforth UB), but policy makers can choose between three different financing forms for UB: a wage tax, a payroll tax and a profit tax while the latter is only paid by exporters. To derive the optimal RS, we first calculate the local optima, i.e. the welfare maximum for each of the three RS. Given this outcome, we find the optimal RS by simply comparing the welfare levels of the local maxima. Hence, we determine the optimal value of the UB and their optimal financial form.

Our model builds on the framework of de Pinto and Michaelis (2011), who combine the Melitz (2003) model of monopolistic competition and heterogeneous firms with the existence of heterogeneous workers (i.e. workers who are different
with respect to their abilities; see Helpman et al., 2010a, b) and unionized labor markets (see Layard/Nickell, 1991).¹ We extend their model to a government sector with the policy instruments mentioned above. To implement a welfare function with the required properties, we use the approaches of Carlsson et al. (2005) and Lommerud et al. (2004). In doing so, we assume that welfare depends positively on aggregate income per capita and negatively on the Gini coefficient, which is the standard measurement of income inequality and can be calculated by observing the Lorenz curve. Additionally, the degree of inequality aversion enters into the welfare function. The government’s optimization problem is to choose a value of UB which maximizes the welfare function subject to the government’s budget constraint. After computing the general equilibrium, we simulate the model with standard calibration values drawn from the relevant literature.

There are four mechanisms driving our results. First, due to the heterogeneity of the firms, the well-known firm selection effect varies the distribution of active firms and thus the average productivity of the firms. Second, there is a firm-specific interval of abilities. Low-productive firms employ low-skilled workers, while firms with relatively high productivity demand and attract workers with relatively high abilities. Third, monopoly unions set a wage rate at the firm level. Maximizing the utility of the unions’ median member yields the Nash-solution: the wage rate is a constant mark-up over the median member’s fallback income. The latter is endogenously derived in our model and turns out to be a positive function of UB and the (worker-specific) net outside wage. Fourth, we assume that only exporters have to pay the profit tax. Since exporters can be approximately treated as winners of trade, we thereby implement a redistribution instrument that directly harms those economic actors who profit from trade liberalization.

Our main findings are: first, if the government chooses RS 1 (UB are financed by a wage tax), the welfare reaction is hump-shaped. In the general equilibrium, we find that the wage tax funding has no impact on welfare (wage tax neutrality). An increase in the wage tax rate leads to both a higher wage mark-up and a lower fallback income, implying that the (gross) wage rate remains constant. In contrast, paying UB decreases aggregate income, since unions set a higher wage rate which raises the unemployment rate. If the value of UB is relatively low, however, income inequality decreases – the income of so far unemployed workers moves up – and this decline is strong enough to overcompensate for the decline in aggregate income. Consequently, welfare increases and reaches a local optimum for a well-defined threshold level of UB. If UB are higher than

¹No doubt, the combination of goods and labor market imperfections as well as the consideration of both, firm and worker heterogeneity imply a highly complex model environment. The modeling approach is though necessary due to several reasons. First, goods and labor market imperfections are needed in models with income redistribution (see Brecher/Choudhri, 1994). Second, using homogeneous firms would ignore the firm-selection effect of trade liberalization which is contrary the empirical evidence. Third, without the existence of heterogeneous workers, a RS cannot capture the implications of skill-specific unemployment rates. Fourth, implementing trade unions instead of other labor market frictions like efficiency wages delivers new channels through which the outcome of the redistribution is highly influenced.
this threshold level, the impact on income inequality becomes weaker, or even negative, because of the increasing unemployment rate; welfare declines. Second, if the government chooses RS 2 (UB are financed by a payroll tax), welfare unambiguously declines. The local optimum is thus the starting position of UB equal to zero. This result is caused by the strong negative impact of the payroll tax on aggregate income.

Third, if the government chooses RS 3 (UB are financed by a profit tax paid by exporters only), the welfare reaction is again hump-shaped. The profit tax unambiguously decreases aggregate income because of its influence on firms’ entry decisions. Paying the profit tax implies a reduction of the average net profit per firm, so market entry becomes less attractive, which implies, in the Melitz framework, a weaker firm-selection; average productivity and thus aggregate income decline. However, the profit tax also unambiguously decreases income inequality. Due to the weaker firm-selection, the proportion of low-productive firms that employ low-skilled workers increases, reducing the unemployment rate and income inequality. Moreover, the number of exporters declines due to our assumption that only exporters pay the profit tax; the employment share of exporters receiving relatively high wages decreases, which also reduces income inequality. Combined with the positive impact of UB on income inequality, the decline in aggregate income is overcompensated for relatively low values of UB. Therefore, welfare increases, reaches a local optimum for a well-defined threshold level of UB and declines after that.

Fourth, comparing the local optima, we obtain an unequivocal ranking for the RS in terms of welfare level: 1. UB financed by a profit tax (paid by exporters only), 2. UB financed by a wage tax and 3. UB financed by a payroll tax. Thus, the optimal RS is to finance UB by a profit tax paid by exporters and to set a value of the UB which leads to the welfare maximum.

Most closely related to our approach is the study of Itskhoki (2008), who addresses the equity-efficiency trade-off caused by the redistribution of trade gains. In his model, firms are worker-entrepreneurs with unobservable agent heterogeneity. Similar to us, a welfare function is implemented where income inequality enters as its negative argument. He argues that, in general, the optimal redistribution policy can be either to increase or to decrease tax rates; the answer depends on the ratio between equity gains and efficiency losses. However, the model focuses only on different tax forms with different degrees of progression, rather than also looking at the implications of a tax-financed government expenditure which benefits low-income workers, such as UB, which is the topic in our approach.

Our paper is also related to the literature that investigates the impact of different RS for trade gains in trade models with heterogeneous firms, monopolistic competition and labor market imperfections. To the best of our knowledge, however, these studies neither exhibit an explicit objective function of the government nor include distributional issues in the welfare function. Egger and Kreickemeier (2009a) introduce a RS that consists of an absolute per capita transfer to all individuals and a proportional profit tax. Using a Melitz style model with fair-wages, they show that there are profit tax rates which equalize
income distribution without eliminating the trade gains completely, i.e. aggregate income does not fall below its autarky level. Egger and Kreickemeier (2012) as well as Helpman and Itskhoki (2010) also consider redistribution in the extended Melitz framework, but the implications of this are only discussed as side-effects. In the former study, which uses fair-wage constraints, UB financed by a proportional income tax decreases employment and welfare. In the latter study, search and matching frictions as well as UB financed by a lump-sum tax, are modeled. In this case, welfare could either increase or decrease and a decrease can be observed for the majority of the parameter constellations.²

The remainder of the paper is structured as follows. In section two, we present the set-up of the open economy model at the sectoral level, while the general equilibrium is derived in section three. In section four, we derive the optimal RS. Section five concludes.

2 Model

2.1 Set-up

Our framework builds on the standard monopolistic competition model with heterogeneous firms by Melitz (2003) and its extension to trade unions and heterogeneous workers by de Pinto and Michaelis (2011).³ We consider an open economy setting with two symmetric countries. The economy consists of two sectors: a final goods sector produces a homogeneous good \( Y \) under perfect competition and a monopolistic competitive sector with \( M \) firms produces a continuum of differentiated intermediate goods.

The production technology of the final goods producer is assumed to be a CES aggregate of all the available intermediate goods:

\[
Y = M_t^{\frac{1}{\sigma}} \left[ \int_{\nu \in V} q(\nu)^{\frac{\sigma-1}{\sigma}} d\nu + \int_{\nu \in V} q_{im}(\nu)^{\frac{\sigma-1}{\sigma}} d\nu \right]^{\frac{\sigma}{\sigma-1}},
\]

\[
P = M_t^{\frac{1}{1-\sigma}} \left[ \int_{\nu \in V} p(\nu)^{1-\sigma} d\nu + \int_{\nu \in V} p_{im}(\nu)^{1-\sigma} d\nu \right]^{\frac{1}{1-\sigma}},
\]

where \( P \) is the corresponding price index. \( V \) denotes the mass of all potentially available goods \( M_t \) and \( \sigma \) represents the elasticity of substitution between any two varieties (\( \sigma > 1 \)). The index \( im \) denotes import variables. Variables without

²In general, our paper can also be classified among the large body of literature concerning the redistribution of trade gains in the absence of firm heterogeneity (see, for instance, Brecher/Choudhri, 1994; Davidson/Matusz, 2006). To the best of our knowledge, however, none of these studies incorporates income inequality into the welfare function and they are thus unable to derive an optimal RS in our sense.

³The same modeling approach is used by de Pinto (2012). Note, however, that his model sticks to the assumption of a welfare function which only depends on aggregate income. Therefore, an optimal RS cannot be derived.
an index refer to the domestic market only. We suppose $Y$ to be the numéraire, which allows for the normalization of the price index: $P \equiv 1$. The demand for variety $v$ can be derived from the profit maximization of the final goods producers:

$$q_t(v) = q(v) + q_{im}(v) = \frac{Y}{M_t} \left[ (p(v))^{-\sigma} + (p_{im}(v))^{-\sigma} \right]. \quad (1)$$

In the intermediate goods sector, there is a continuum of ex ante homogeneous firms. Firms enter the differentiated sector by paying a fixed entry cost $f_e > 0$ (measured in units of final goods and equal for all firms). $f_e$ can be interpreted as the irreversible investment for research and development which all firms have to incur. After it has been paid, $f_e$ is sunk. In the subsequent Melitz lottery, firms observe their entrepreneurial productivity $\phi$, which is Pareto distributed with $G_\phi(\phi) = 1 - (\phi_{\text{min}}/\phi)^k$ for $\phi \geq \phi_{\text{min}} = 1$ and $k > 1$. In addition to the entry cost, there are fixed production costs $f > 0$ and $f_x > 0$ (measured in units of final goods and equal for all firms). $f$ and $f_x$ can be interpreted as the costs of forming a distribution and servicing network in the domestic and foreign markets, respectively. These types of fixed costs are called beachhead costs.

The economy is endowed with an exogenous number of heterogeneous workers $L$, who differ in their abilities $a_j$, $j = 1, \ldots, L$. The worker’s abilities are drawn from a Pareto distribution $G_a(a) = 1 - (a_{\text{min}}/a)^k$ for $a \geq a_{\text{min}} = 1$ and individuals are assumed to know and maintain their ability levels at any point in time.\footnote{Helpman et al. (2010a, b) introduced this concept in order to allow for worker heterogeneity. However, in their model, abilities are match-specific and independently distributed. Hence, a worker’s ability for a given match does not convey any information about his or her ability for other (future) matches. The ability of an individual worker is unobservable, even if the worker has an “employment history”.}

Besides firms and workers, there is a government sector. On the expenditure side, the government pays (worker-specific) UB $B_j$. On the revenue side, three kinds of taxes are distinguished: a proportional wage tax $T_w$, a proportional payroll tax $T_{pw}$ and a proportional profit tax $T_\pi$. We assume that the latter is paid by exporters $M_x$, where the tax base is the exporters’ total profit, $\pi + \pi_x$, i.e. the sum of domestic and export profits. Since exporters receive an increase in market share from trade liberalization, the government thus has a policy instrument to direct harm the winners of trade.\footnote{Note that this approach is a short-cut for different profit tax rates for firms with different productivities. To keep our analysis as simple as possible, we set the profit tax rate for non-exporting firms at zero while assuming a positive value for exporting firms.}

The corresponding proportional tax rates are $t_w \in (0, 1)$, $t_{pw} \in (0, 1)$ and $t_\pi \in (0, 1)$.

Let us now turn to the firms’ production technology. Consider a firm $i$ with

$\phi$ is slightly different to that of Melitz (2003). We prefer the term entrepreneurial (instead of firm) productivity, in order to distinguish between the quality of the management and originality of the business idea, on the one hand, and a firm’s total productivity, which also depends on the quality of its employed workers, on the other.
productivity $\phi_i$. The production technology is given by:

$$q_i = h_i \phi_i \overline{\pi}_i,$$

where $h_i$ denotes the number of employees and $\overline{\pi}_i$ represents the average ability of employees. A firm does not demand all abilities but sets a minimum quality requirement. This minimum quality requirement is firm-specific, and it increases with entrepreneurial productivity $\phi$. For concreteness, we assume:

$$a^*_i = \phi_i^\alpha \quad \text{with} \quad \alpha \geq 0.$$  

Eq. (3) represents a firm’s technology constraint: firm $i$ does not employ workers with abilities lower than $a^*_i$ because its marginal product of labor is zero (or even negative because of complementarities, see Helpman et al., 2010a, b). Parameter $\alpha$ denotes the sensitivity of $a^*_i$ with respect to entrepreneurial productivity.$^7$

The wage offer matters. Just as a firm might not want to hire a low-ability worker, a worker might not want to work for a low-wage firm. Individuals differ with respect to their reservation wages. The higher the ability of an individual, the higher is the marginal product of labor, and the higher is the reservation wage. A worker does not apply for jobs paying less than his/her reservation wage.

As a result, we can identify an upper bound of abilities for each firm. If firm $i$ offers a wage rate $w_i$, there will be a worker who is indifferent between (short-term) unemployment and employment in firm $i$. We define this worker as employee $z_i$ with ability $a_{z_i}$ and reservation wage $b_{z_i}$. The indifference condition is given by $w_i^{\text{net}} = (1-t_w)w_i = b_{z_i}$. For the wage offer $w_i$, firm $i$ attracts workers with abilities $a \leq a_{z_i}$, workers with $a > a_{z_i}$ do not apply for a job in firm $i$.

The abilities of firm $i$’s employees lie within the interval $a^*_i$ and $a_{z_i}$, where the limits depend on the productivity $\phi_i$ and the wage rate $w_i$. The average ability of the firm-specific interval is given by (see de Pinto/Michaelis, 2011 for the derivation):

$$\overline{\pi}_i = \Gamma_1 \frac{(a^*_i)^{1-k} - (a_{z_i})^{1-k}}{(a^*_i)^{-k} - (a_{z_i})^{-k}} \quad \text{with} \quad \Gamma_1 = \frac{k}{k-1}.$$  

The determination of employment and wages at the sectoral level is modeled as a five-stage game, which we solve by backward induction. In the first stage, firm $i$ participates in the Melitz lottery and discovers its entrepreneurial productivity $\phi_i$. Given $\phi_i$, firm $i$ decides whether to produce or not and, additionally, whether to export or not. In the case of production, firm $i$ posts a vacancy (stage two). The job description includes the minimum quality requirement $a^*_i$ and a wage offer $w_i$, where we insinuate that firms anticipate correctly the outcome of the wage setting in stage four. Therefore, the offered wage is identical to the paid wage $w_i$. Additionally, posting a vacancy is assumed to be costless. More precisely, the advertisement does not create variable costs.

$^7$The minimum quality requirement assumption can be motivated from both an empirical and a theoretical point of view. For a detailed discussion, see de Pinto and Michaelis (2011).
In the third stage, workers collect information about job vacancies. Information gathering is costless, so that all workers have perfect knowledge of all job descriptions. If the marginal costs of applications are zero, the optimal strategy of a worker $j$ with ability $a_j$ is to apply for all jobs with a minimum quality requirement $a_j^* \leq a_j$ and a (net) wage offer no less than his or her reservation wage. Any firm $i$ thus obtains a full distribution of abilities between the limits $a_i^*$ and $a_i$. To extract an economic rent, the applicants form a monopoly trade union at the firm level. The membership of monopoly union $i$ is denoted by $n_i$. Note that a worker will only apply for those vacancies s/he expects s/he will accept. Consequently, a worker accepts the offer of any job for which s/he has applied (see Layard et al., 1991).

In the fourth stage, the monopoly union $i$ sets the wage rate $w_i$, where the employment decision of the firm in stage five is anticipated. After the firm has set the optimal employment level $h_i$, it randomly draws workers from among the union members until $h_i$ is reached. Since all union members fulfill the minimum quality requirement and the union members accept the job offer, there will be a “drawing without repetition”. We abstract from a (costly) screening technology. Firms are assumed to be able to observe the minimum ability of a worker at no cost, but they are not able to observe the exact value of $a_j$ of an individual worker. Furthermore, note that the existence of unions eliminates any wage differentiation within firms.

### 2.2 Labor demand

To solve the five-stage game, we continue to focus on firm $i$ with entrepreneurial productivity $\phi_i$. Firm $i$ can either serve the domestic market only or can additionally export goods abroad. We first look at firm $i$’s optimal behavior in the domestic market and take up the endogenous export decision afterwards (see section 2.4).

We begin by discussing the derivation of the labor demand at stage five, where $w_i$, $a_z$, $a_i^*$ and $a_i$ are already determined. Each firm faces a constant elasticity demand curve (1), which leads to firm’s revenues:

$$r_i = q_i^\sigma (Y/M_t)^{1/\sigma}, \quad \kappa \equiv 1 - \frac{1}{a},$$

(5)

where $\kappa$ denotes the degree of competitiveness in the market for intermediate goods. The firm maximizes net profits by setting employment such that the marginal revenue of labor equals the marginal costs: $\partial r_i / \partial h_i = (1 + t_{pw})w_i$. The optimal level of employment is given by:

$$h_i = \left( \frac{\kappa \phi_i^\sigma a_i^\sigma}{(1 + t_{pw})w_i} \right)^{\sigma} \frac{Y}{M_t}.$$  

(6)

As usual, the firm’s labor demand curve is negatively sloped in the $(h, w)$-space.\(^8\) Note that the number of available goods $M_t$ and aggregate output $Y$ are exogenous at the sectoral level.

\(^8\)In our model, this outcome is, however, not trivial. A wage hike swells the firm-specific
The optimal price

\[ p_i = \frac{1}{\kappa} \frac{(1 + t_{pw})w_i}{\phi_i \bar{a}_i} \]  

(7)

is a constant mark-up \( 1/\kappa \) over marginal costs. Note that \( p_i \) is independent of the profit tax rate \( t_p \). Every price setting that implies profit maximization before the profit tax remains also optimal after the profit tax as long as the profits are still positive. To complete our analysis of stage five, we reformulate the firm’s revenue as a function of its optimal price setting:

\[ r_i = p_i^{1-\sigma} \frac{Y}{M_t}. \]  

(5’)

2.3 Union wage setting

In the fourth stage, the monopoly union \( i \) sets the wage rate \( w_i \), at which the number of union members \( n_i \) is already fixed. As shown above, union members are heterogeneous with respect to their abilities, which lie within the interval \( a_i^a \) and \( a_i^z \). The monopoly union maximizes the expected utility of the median member \( m_i \) (see Booth, 1984), and thus the objective function is given by:

\[ EU_{m_i} = \frac{h_i}{n_i} (1 - t_w) w_i + \left(1 - \frac{h_i}{n_i}\right) b_{m_i}, \]  

(8)

with \( b_{m_i} \) denoting the reservation wage (fallback income) of the median member. Note that membership \( n_i \) exceeds the firm’s labor demand \( h_i \) because of the game structure at stage three (see below). Furthermore, the monopoly unions are risk-neutral by assumption.

The monopoly union \( i \) fixes \( w_i \) to maximize the Nash product \( NP_i = EU_{m_i} - U_{m_i} \) subject to \( \partial r_i/\partial h_i = (1 + t_{pw})w_i \), with \( U_{m_i} = b_{m_i} \) being the union’s fallback position. Owing to the constraint, the union anticipates that the firm chooses a point on its labor demand curve for any given \( w_i \). The solution of the optimization problem leads to a well-known result: the wage \( w_i \) is a mark-up \( (1 + t_{pw})w_i \) over the median member’s fallback income:

\[ w_i = \frac{\theta}{1 - t_w} b_{m_i} \]  

(9)

with \( b_{m_i} \) denoting the reservation wage (fallback income) of the median member. Consequently, there are two effects operating in opposite directions in response to a wage increase: marginal costs and marginal revenues both shift up. The strength of the latter effect can be measured by the wage elasticity of average abilities \( \epsilon_{a_i^w, w_i} \). As shown in detail by de Pinto and Michaelis (2011), \( \epsilon_{a_i^w, w_i} \) is equal across all firms and (for reasonable parameter settings) smaller than one. Then, the derivation of (6) with respect to \( w_i \) proves that \( \partial h_i/\partial w_i < 0 \) holds for \( \epsilon_{a_i^w, w_i} < 1 \). Increasing marginal revenue does not compensate for rising marginal costs, but it mitigates the employment reduction.

Recall that the labor demand curve becomes steeper if the wage rate increases because of rising average abilities. Consequently, the monopoly union also anticipates the positive effect of a higher wage rate, but, as shown above, employment nevertheless decreases.
The union generates an economic surplus for its members, which we define as the difference between the wage rate \( w_i \) and the fallback income of the median member \( b_{m_i} \). The wage rate \( w_i \) is increasing c.p. in the wage tax \( t_w \), reflecting the unions’ aim of stabilizing workers’ net wages.

We complete the analysis of stage four by the derivation of the fallback income of worker \( j \) with ability \( a_j \). If worker \( j \) is the median member of firm \( i \), we have \( j = m_i \). Following the step-by-step derivation of de Pinto (2012) with the use of value functions, we obtain:

\[
b_j = u B_j + (1 - u)(1 - t_w)\bar{w}_j. \tag{10}
\]

As mentioned, the fallback income of worker \( j \) corresponds to the reservation wage of worker \( j \). The reservation wage is increasing in the UB, \( B_j \), and increasing in the outside wage \( \bar{w}_j \), which is defined as \( j \)'s expected wage rate in the economy.

Let us have a closer look at the outside wage. The empirical literature shows that wages are determined by both individual characteristics and a country’s macroeconomic performance (see, for instance, Fairris/Jonasson, 2008; Holmlund/Zetterberg, 1991; Nickell/Kong, 1992). We take up this observation by assuming that the outside wage is a convex combination of a microeconomic and a macroeconomic variable:

\[
\bar{w}_j = (a_j)\omega \left( w(\bar{\phi}_t) \right)^{1-\omega} \quad 0 \leq \omega \leq 1. \tag{11}
\]

In our context, the most plausible microeconomic variable is the ability \( a_j \) of worker \( j \). The higher the skill level of a worker, the higher is the wage s/he can expect in the economy (or: the computer scientist expects a higher wage than the collector irrespective of the state of the economy). Less obvious is the macroeconomic variable. In a world with homogeneous workers, where, by definition, individual characteristics do not matter (\( \omega = 0 \)), consistency requires that the outside wage coincides with the wage prevailing in a (symmetric) general equilibrium (see, for instance, Layard/Nickell, 1990). We pick up this scenario by assuming that the outside wage of a worker \( j \) is increasing in the wage rate, which holds in the general equilibrium, \( w(\bar{\phi}_t) \), where \( \bar{\phi}_t \) denotes the entrepreneurial productivity of the representative firm (see below).\(^{10}\)

The UB of worker \( j \) are modeled as a constant share of his/her net outside wage:

\[
B_j = s (1 - t_w) \bar{w}_j, \tag{12}
\]

\(^{10}\)One might argue that high-skilled workers with a reservation wage above the wage paid by the representative firm are not affected by \( w(\bar{\phi}_t) \). Consequently, \( w(\bar{\phi}_t) \) should not be part of their outside option. However, in a Melitz world with Pareto-distributed productivities, the aggregate variables have the property that they are identical to what they would be if the economy were endowed with \( M_t \) identical firms with productivity \( \bar{\phi}_t \). Therefore, \( w(\bar{\phi}_t) \) is only a shortcut for the “true” distribution of wages in the economy. A shift in \( w(\bar{\phi}_t) \) should thus be interpreted as a proxy for a shift in the whole wage distribution, thus affecting all wages irrespective of skill level.
with $0 \leq s \leq 1$ denoting the replacement ratio that is set by the government. Eq. (12) fits two important properties concerning the design of UB. First, $B_j$ is worker-specific. High-skilled workers (computer scientists) exhibit a higher outside wage and thus receive a higher benefit than low-skilled workers (collectors). Thus, UB depend on the worker’s employment history. Second, $B_j$ is a positive function of the country’s macroeconomic performance, reflecting the connection between government expenditure and the business cycle (for a similar modeling approach, see Haan/Prowse, 2010 and, for empirical evidence, see Fitzenberger/Wilke, 2010).

With these building blocks in place and noting $j = m_i$, the fallback income $b_{m_i}$ and the bargained wage $w_i$ can be rewritten as:

$$b_{m_i} = (1 - t_w)(1 - u(1 - s))(a_{m_i})^\omega \left( w(\bar{\phi}_i) \right)^{1-\omega},$$  \hspace{1cm} (13)$$

$$w_i = \theta (1 - u(1 - s))(a_{m_i})^\omega \left( w(\bar{\phi}_i) \right)^{1-\omega},$$  \hspace{1cm} (14)$$

respectively. Note that owing to heterogeneous individuals, the economic surplus (bargained wage minus reservation wage) differs between union members. Within the firm’s and the union’s ability interval, the worker with the minimum qualification obtains the largest rent (lowest reservation wage). The surplus declines with members’ ability levels, because of an increasing reservation wage. Member $z_i$ with the highest qualification has a zero surplus, which makes him or her indifferent between taking a job in firm $i$ and looking for a job elsewhere.

### 2.4 Union membership, vacancy posting and the Melitz lottery

Stage three determines union membership $n_i$. As illustrated above, all workers with ability $a^*_i \leq a \leq a_{z_i}$ apply for a job at firm $i$, so that each firm $i$ gets the full distribution of abilities within the two limits. Workers with an ability greater than $a_{z_i}$ have a reservation wage exceeding $w_i$, so they do not apply and they are not members of monopoly union $i$. The number of applicants and thus the number of union members is given by:

$$n_i = \int_{a^*_i}^{a_{z_i}} ka^{-(1+k)} da = (a^*_i)^{-k} - (a_{z_i})^{-k}.$$  \hspace{1cm} (15)$$

As shown by de Pinto and Michaelis (2011), the ability level of the median member can be derived as:

$$a_{m_i} = 2^{1/k} \left[ (a_{z_i})^{-k} + (a^*_i)^{-k} \right]^{-1/k}.$$  \hspace{1cm} (16)$$

In order to determine the ability limits we turn to the posting of the vacancy, which is the topic of stage two, where a firm’s entrepreneurial productivity $\phi_i$
is already predetermined. The lower limit is obviously given by the minimum ability requirement, $a_i^* = \phi_i^*$. The upper limit, by contrast, is determined by the requirement that the posted net wage equals the reservation wage of the efficient worker $z_i$: $(1 - t_w)w_i = b_{z_i}$. As shown by de Pinto (2012), with this condition we can calculate the ability level of firm $i$’s efficient worker:

$$a_{z_i} = A^{1/k} \phi_i^* \quad A = 2\theta^k/\omega - 1. \quad (17)$$

Inserting (3) and (17) into (4) yields:

$$\bar{a}_i = \Gamma_1 \Gamma_2 \phi_i^2, \quad \Gamma_2 = \frac{A - A^{1/k}}{A - 1}. \quad (18)$$

Moreover, substituting (3) and (17) into (16) and observing (14), we can rewrite the wage rate as:

$$w_i = A^{\omega/k} \left(1 - u(1 - s)\right) w(\bar{\phi}_i)^{1-\omega} \phi_i^{\omega}. \quad (19)$$

In stage one, firm $i$ participates in the Melitz lottery and draws the entrepreneurial productivity $\phi_i$. Subsequently, the firm has to decide whether to enter the domestic market and to produce or not as well as whether to serve the foreign market and to export or not. A firm will produce for the domestic market if and only if the drawn entrepreneurial productivity is at least as high as the cut-off productivity level $\phi^*$: $\phi_i \geq \phi^*$. In this case, the expected stream of profits is non-negative. The firm with the lowest possible productivity $\phi^*$ is called the marginal firm.

Concerning the export decision, there are variable iceberg costs $\tau \geq 1$ besides the already mentioned beachhead cost $f_x \geq 0$. Furthermore, exporting creates a third cost component, i.e. the profit tax on domestic profits $t_\pi \pi_i$, which is zero if firm $i$ does not export due to our assumption that the profit tax is only borne by exporters. There is an export cut-off level $\phi_x^*$ such that for $\phi_i \geq \phi_x^*$ the additional revenue from exporting is at least as high as the additional costs. In line with Melitz (2003), only a fraction of firms engage in exporting. For $\phi_i \geq \phi_x^*$, firms are exporters and produce for both the home and the foreign market. For $\phi^* \leq \phi_i < \phi_x^*$, firms produce for the home market only.

If firm $i$ draws a productivity that exceeds or at least equals the export cut-off level, $\phi_i \geq \phi_x^*$, the derivation of the corresponding export values is needed. The net export profit is defined by $\pi^\text{net}_{ix} \equiv (1 - t_\pi)(r_{ix}/\sigma - f_x - t_\pi \pi_i)$. Profit maximization yields $p_{ix} = \tau p_i$, $q_{ix} = \tau^{-\sigma} q_i$, $h_{ix} = \tau^{1-\sigma} h_i$ and $r_{ix} = $.
Thus, the export variables can be expressed as a function of the domestic variables (see also Melitz, 2003). Using the simplifying assumption of \( f = f_x \) (see Egger/Kreickemeier, 2009b for a justification) and \( \pi_{\text{net}}^i = (1 - t_{\pi})(\frac{R_i}{R} - f) \), which follows from \((5')\), we can reformulate the net export profit:

\[
\pi_{\text{net}}^i = (1 - t_{\pi}) \left( (\tau^{1-\sigma} - t_{\pi}) \frac{R_i}{R} - (1 - t_{\pi}) f \right).
\] (20)

Finally, the existence of the marginal firm with productivity \( \phi^* \) has important consequences for the segregation of the labor force of the economy. Analogous to firm \( i \), the marginal firm also sets a minimum quality requirement \( a^* \). Since no firm has a lower entrepreneurial productivity, \( a^* \) can be interpreted as the minimum quality requirement for the whole economy. For workers with \( a < a^* \), their abilities are not sufficient to gain any job, as no active firm on the market will demand qualifications below \( a^* \). With (3), we obtain:

\[
a^* = (\phi^*)^\alpha.
\] (21)

Thus, we divide the labor force \( T \) into two groups: (i) active\(^{13}\) workers \( L \) with \( a \geq a^* \) and \( u = 1 - \frac{H}{L} < 1^{14} \) and (ii) (long-term) unemployed persons \( L' \) with \( a < a^* \) and \( u' = 1 \). The latter will never be members of a union because they are not able to meet the job requirements. Consequently, the monopoly union only accounts for active workers in the wage-setting process.

Long-term unemployed persons also receive UB. In contrast to the UB of active workers, we eliminate the worker-specific component. The reason is simple. Since a person with an ability below \( a^* \) has no opportunity to get a job in the economy, her/his outside wage drops to zero and according to (12) the UB would be zero as well. To avoid this, we assume that the UB of long-term unemployed persons is a constant share \( s \) of the net equilibrium wage rate instead of the worker-specific net outside wage. Formally, we get:

\[
B^*_j = s(1 - t_w)w(\tilde{\phi}_j) \quad \text{if} \quad j \in [1, a^*).
\] (22)

Notably, Eq. (22) is a special case of the general formulation in (12) which holds if the microeconomic variable in the outside wage disappears \( (\omega = 0) \).

To complete our model at the sectoral level, we look at the relationship between two firms with different entrepreneurial productivities, \( \phi_1 \) and \( \phi_2 \). From trade costs from \( \pi_{xy} \). Second, we avoid a discontinuity in the export profit function. If \( t_{\pi} \pi_j \) disappears, firms with a positive export profit up to a certain threshold have no incentive to export because of the profit tax on domestic profits. Note again that the profit tax base is the exporter’s total profit. Consequently, not only \( t_{\pi} \pi_{\text{net}} \) but also \( t_{\pi} \pi_{\text{net}} \) has to be considered for the definition of net export profit.

\(^{13}\)“Active” means that these workers have a positive employment probability. Nevertheless, at any point in time a fraction of active workers is unemployed.

\(^{14}\)Notably, entrepreneurial productivity and workers’ abilities are both Pareto-distributed with identical lower bounds and shape parameter \( k \). These characteristics, combined with the assumption of random matching, imply that the ratio of employed workers with ability \( j \), \( H_j \), to the number of all workers with ability \( j \), \( L_j \), is equal for all \( j \). As a result, the unemployment rate is identical across all abilities: \( u = u_j = 1 - \frac{H_j}{L_j} \forall j \).
(5'), (7), (18) and (19), we obtain:

\[
\frac{r(\phi_1)}{r(\phi_2)} = \left(\frac{\phi_1}{\phi_2}\right)^\beta \quad \beta \equiv (\sigma - 1)(1 + \alpha - \alpha \omega) > 0.
\] (23)

Assuming \( \phi_1 > \phi_2 \), this shows that firms with higher entrepreneurial productivities generate higher revenues than low-productive firms. In the same way, we can calculate:

\[
h(\phi_1) = \left(\frac{\phi_1}{\phi_2}\right)^{\beta - \alpha \omega},
\] (24)

where it is not clear-cut whether high-productive firms employ more workers than low-productive firms (for a detailed discussion of this issue see Egger/Kreichemier, 2009b).

### 2.5 The government’s objective function

Let us now turn to the goals of the government in more detail. Observing the implications of trade liberalization, i.e. a simultaneous increase in aggregate income and income inequality (see section 4), the government aims to redistribute income in order to gain political support for trade liberalization and to maximize the probability of reelection.

A common approach in the literature is to assume that the government’s objective function is identical with the welfare function. However, in most of the existing studies, welfare is only measured by aggregate income per capita – the tradition welfare measurement. Due to the well-known equity-efficiency trade-off, the government can thus only design an RS which minimizes the decline in aggregate income for an ex ante given level of equity (or equivalently which maximizes equity for an ex ante given level of aggregate income).

In our model, we also assume that the government’s objective function is described by the welfare function but we use a different approach in calculating the latter. In line with Carlsson et al. (2005) and Lommerud et al. (2004), welfare accounts for both aggregate income and distribution of income. Adopting their approaches for our model, welfare and thus the government’s objective function is given by:

\[
WF = \frac{I}{L} \Theta^{-\gamma} \quad 0 \leq \Theta \leq 1,
\] (25)

with \( \frac{I}{L} \) denoting aggregate income per capita and \( \Theta \) representing the Gini coefficient as the standard measure of income inequality. \( \gamma \in [0, 1] \) measures the income inequality aversion and can be interpreted as an inequality elasticity that indicates the percentage change in income for a 1% rise in inequality in order to hold utility constant. Note that if \( \Theta \) increases, income inequality rises and welfare declines.\footnote{The welfare function is rather unconventional. For a detailed motivation for it and a discussion of its properties see Lommerud et al. (2004).}
3 General equilibrium

So far, we have described the model at the sectoral level and introduced the government sector. To find the optimal RS, we additionally have to calculate the general equilibrium and to specify the government’s decision problem which are the topics of this section.

3.1 Average productivity and aggregation

Consider first the weighted average productivity level of all active firms in a country $\tilde{φ}_t$. By following the step-by-step derivation of Egger and Kreickemeier (2009b), we get:

$$\tilde{φ}_t = \tilde{φ} \left[ \frac{1}{1 + \chi} \left( 1 + \chi^{1-\sigma} \left( \frac{\tilde{φ}_x}{\tilde{φ}} \right)^{\alpha} \right) \right]^{1/\beta}, \quad (26)$$

where $\chi$ denotes the ex ante probability of being an exporter:

$$\chi = \frac{1 - G_\phi(\phi^*_x)}{1 - G_\phi(\phi^*)} = \left( \frac{\phi^*}{\phi^*_x} \right)^k, \quad 0 \leq \chi \leq 1. \quad (27)$$

$\tilde{φ}$ is the average productivity of all domestic firms and $\tilde{φ}_x$ is the average productivity of exporting firms. Owing to the Pareto distribution, these productivities are given by:

$$\tilde{φ} = \xi_1^{1/\beta} \phi^*, \quad (28)$$

$$\tilde{φ}_x = \xi_1^{1/\beta} \phi^*_x, \quad (29)$$

$$\xi_1 = \frac{k}{k - \beta} \quad \text{with} \quad k > \beta.$$

The inspection of (27), (28) and (29) indicates that the total average productivity $\tilde{φ}_t$ depends on the relation between the export cut-off level $\phi^*_x$ and the cut-off productivity level $\phi^*$. To calculate $\phi^*_x / \phi^*$ (and hence $\phi_x / \tilde{φ}$), we use the well-known zero cut-off profit condition (henceforth ZPC) (see Melitz, 2003). By definition, the marginal firm $\phi^*$ gains a zero net profit: $\pi(\phi^*) = 0$: \footnote{Notably, (27) implies $\phi^* < \phi^*_x$. Thus, the marginal firm only produces for the domestic market, concluding $\pi^{net}(\phi^*) = \pi(\phi^*) = \frac{1}{2}r(\phi^*) - f$.}

$$r(\phi^*) = \sigma f. \quad (30)$$

By analogy, we define $\pi^{net}(\phi^*_x) = 0$, for where a firm just breaks even even in the export market. This condition holds if and only if the exporting revenue covers the extra trading costs. From (20) and $\phi_i = \phi^*$, we get:

$$r(\phi^*_x) = \sigma f \Omega \quad \Omega \equiv \frac{1 - t_x}{r^{1-\sigma} - t_x} > 1, \quad (31)$$
with $\tau^{1-\sigma} > t_\pi$ by assumption.\textsuperscript{17} Additionally, we can define the productivity distribution of firms in the market, $\mu(\phi)$ and the productivity distribution of exporting firms, $\mu_x(\phi)$:

\[
\mu(\phi) = \begin{cases} \frac{g_\sigma(\phi)}{1-\sigma_x(\phi^*)} = k \left( \frac{\phi^*}{\phi} \right)^k & \text{if } \phi \geq \phi^* \quad \text{and} \\ 0 & \text{otherwise} \end{cases},
\]

\[
\mu_x(\phi) = \begin{cases} \frac{g_\sigma(\phi)}{1-\alpha_x(\phi^*)} = k \left( \frac{\phi^*}{\phi} \right)^k & \text{if } \phi \geq \phi_x^* \\ 0 & \text{otherwise} \end{cases}.
\]

Combining (30) and (31) with (23) and using (28) as well as (29) leads to:

\[
\left( \frac{\phi_x^*}{\phi} \right)^\beta = \left( \frac{\phi^*}{\phi} \right)^\beta = \Omega.
\] (34)

Next, we combine (27) with (34) to get:

\[
\chi = \Omega^{-k/\beta}.
\] (26')

Substituting (34) into (26) and using (26'), we finally obtain:

\[
\overline{\phi}_t = \overline{\phi} D,
\] (35)

\[
D \equiv \left( \frac{\Delta_1}{1 + \chi} \right)^{1/\beta} \geq 1 \quad \text{and} \quad \Delta_1 \equiv 1 + \tau^{1-\sigma} \chi^{(k-\beta)/k}.
\]

The difference between the two averages $\overline{\phi}_t$ and $\overline{\phi}$ can be explained by the interplay between the lost-in-transit effect (henceforth LT), i.e. goods vanish en route because of iceberg transport costs and the export-selection effect (henceforth ES), i.e. exporting firms are the most productive in the economy. With $t_\pi > 0$, we observe that ES is greater than LT, which implies an increase in total average productivity $\overline{\phi}_t$. This mechanism is represented by the parameter $D$ in Eq. (35) (see de Pinto, 2012 for a more detailed discussion concerning ES).

The aggregate variables are derived in the standard way with the underlying assumption of an equalized balance of payments. It follows: $P = p(\overline{\phi}_t) \equiv 1$, $Y = M_t q(\overline{\phi}_t)$ and $R = M_t r(\overline{\phi}_t)$. The aggregate gross profit is calculated for the hypothetical case that the profit tax is withheld by exporters. We obtain the standard formulation $\Pi = M_t \pi(\overline{\phi}_t)$ (see Melitz, 2003). For the employment level, we get:

\[
H = M h(\overline{\phi}_t) \cdot \xi_1^{\omega/k} \xi_2 \psi_1,
\] (36)

\[
\xi_2 \equiv \frac{k - \beta}{k - \beta + \alpha \omega}, \quad \psi_1 \equiv D^{\omega \beta - \beta} \Delta_2, \quad \Delta_2 \equiv 1 + \tau^{1-\sigma} \chi^{(\omega + k - \beta)/k}.
\]

\textsuperscript{17}Note that if all firms pay the profit tax, the export decision is independent of $t_\pi$ and we obtain $r(\phi_x^*) = \sigma f \tau^{\sigma - 1}$. 

16
Recall that $M_x$ represents the number of exporters and $M$ denotes the number of firms located in a country. The total number of all active firms (and thus the number of all available varieties) in a country is given by $M_t = M + M_x = M(1 + \chi)$.

The aggregate (total) unemployment rate $\bar{u}$ is a weighted average of $u^l$ and $u$. Using the probabilities $P(a < a^*) = 1 - (a^*)^{-k}$ and $P(a > a^*) = (a^*)^{-k}$ as weights yields $\bar{u} = u^l \frac{L^l}{L} + u^u \frac{L^u}{L} = 1 \cdot (1 - (a^*)^{-k}) + u \cdot (a^*)^{-k} = 1 - (1 - u) (a^*)^{-k}$. Noting that $u = 1 - H/L$, the aggregate unemployment rate simplifies to:

$$\bar{u} = 1 - (a^*)^{-k} \frac{H}{L}. \quad (37)$$

The aggregate variables have an important property (see Melitz, 2003): the aggregate levels of $P$, $Y$, $R$, and $H$ are identical to what they would be if the economy were endowed with $M_t$ identical firms with productivity $\tilde{\phi}_t$. Therefore, we treat the firm with productivity $\tilde{\phi}_t$ as the representative firm for the economy. Note that the equations for $P$, $Y$, $R$, and $H$ are aggregation rules. To determine their levels in the equilibrium, we have to add the firm entry and exit conditions and the labor market clearing condition.

Turning to the government sector, we calculate the aggregate levels of $UB$, the wage tax, the payroll tax and the profit tax (see Appendix A for the analytical evidence):

$$B = B^l + B^u = s(1 - t_w) \left[ w(\tilde{\phi}_t) L^l + \xi_3 \cdot (a^*)^\omega w(\tilde{\phi}_t) 1 - \omega u L \right], \quad (38)$$

$$T_w = t_w W, \quad (39)$$

$$T_{pw} = t_{pw} W, \quad (40)$$

$$T_\pi = t_\pi M_x \left( \pi(\tilde{\phi}_x) + \pi_x(\tilde{\phi}_x) \right), \quad (41)$$

where $\xi_3$ is a constant defined in Appendix A and $W$ denotes the aggregate wage income. With (41) at hand, the aggregate net profit is given by:

$$\Pi^{net} = M_t \pi(\tilde{\phi}_t) - t_\pi M_x \left( \pi(\tilde{\phi}_x) + \pi_x(\tilde{\phi}_x) \right). \quad (42)$$

### 3.2 Firm entry and exit

We now turn to the analysis of firm entries and exits, which ends up in the determination of the cut-off productivity $\phi^*$. In line with Melitz (2003), two conditions must hold in the case of production: the free entry condition (henceforth FE) and the ZPC.

We have already introduced the ZPC and obtained (30). In a next step, we derive the average net profit per firm $\pi_t^{net} \equiv \Pi^{net}/M$. Using (42), $M_t = M(1 + \chi)$ and $M_x = \chi M$ yields:

$$\pi_t^{net} = (1 + \chi) \pi(\tilde{\phi}_t) - t_\pi \chi (\pi(\tilde{\phi}_x) + \pi_x(\tilde{\phi}_x)). \quad (43)$$
Substituting $r\tilde{\phi}_t = (\tilde{\phi}_t/\phi^*)^\beta r(\phi^*)$ as well as $r\tilde{\phi}_x = (\tilde{\phi}_x/\phi^*)^\beta r(\phi^*)$ into $\pi(\tilde{\phi}_t) = r(\tilde{\phi}_t)/\sigma - f$, $\pi(\tilde{\phi}_x) = r(\tilde{\phi}_x)/\sigma - f$ and $\pi_x(\tilde{\phi}_x) = r^{1-\sigma}(\tilde{\phi}_x)/\sigma - f$, respectively and observing (30), (28), (35) and (31) leads to $\pi(\tilde{\phi}_t) = (D^\beta \xi_1 - 1)f$, $\pi(\tilde{\phi}_x) = (\chi^{-\beta/k}\xi_1 - 1)f$ and $\pi_x(\tilde{\phi}_x) = (\tau^{1-\sigma}\chi^{-\beta/k}\xi_1 - 1)f$. Inserting these expressions into (43), we finally obtain the average net profit in the presence of the ZPC:

$$\pi_{t}^{\text{ext}} = (1 + \chi)(D^\beta \xi_1 - 1) f - \chi t_\pi (1 + \tau^{1-\sigma})\chi^{-\beta/k}\xi_1 - 2)f. \quad (44)$$

As a result, the average net profit $\pi_{t}^{\text{ext}}$ in the economy is independent of $\phi^*$, which is a direct consequence of the Pareto distribution properties. Obviously, the aggregate net profit $\Pi^{\text{ext}} = M\pi_{t}^{\text{ext}}$ depends in addition on the number of firms operating in the market.

The FE ensures that all existing firms have an incentive to participate in the Melitz lottery. Formally, this requires $f_e = (1 - G_\phi(\phi^*))\pi_{t}^{\text{ext}}/\delta$, with $1 - G_\phi(\phi^*)$ denoting the probability of a successful draw and $\delta$ representing the exogenous death probability of firms. Hence, in the equilibrium, the sunk cost component is equal to the expected discounted average net profits. Using the Pareto distribution, we obtain:

$$\pi_{t}^{\text{ext}} = (\phi^*)^k \delta f_e. \quad (45)$$

With (44) and (45) at hand, we compute the cut-off productivity level:

$$\phi^* = \left[ \left( \frac{1 + \chi}{\delta f_e} \left( D^\beta \xi_1 - 1 \right) - \chi t_\pi (1 + \tau^{1-\sigma})\chi^{-\beta/k}\xi_1 - 2 \right) \right]^{1/k}. \quad (46)$$

The formulation in (46) fits two special cases that can be found in the literature. First, if there is no profit tax, we have $t_\pi = 0$ and $D = 1$, the cut-off productivity drops to $\phi^*_1 = \left[ (1 + \chi)(\xi_1 - 1) f/\delta f_e \right]^{1/k}$ (see Egger/Kreickemeier, 2009b for the same result). Second, if all firms (not just exporters) have to pay the profit tax, $\chi = \tau^{-(\sigma - 1)k/\beta}$, $D = 1$ and $\tilde{\phi} = \tilde{\phi}$ holds because the export cut-off is then independent of $t_\pi$. Immediately, (43) changes to $\pi_{t}^{\text{ext}} = (1 + \chi)\pi(\tilde{\phi}_t) + \chi \pi_x(\tilde{\phi}_x)$. It can be easily shown that $\pi(\tilde{\phi}_t) + \chi \pi_x(\tilde{\phi}_x)$ is equal to $(1 + \chi)\pi(\tilde{\phi}_t)$, which implies $\phi^* = \left[ (1 + \chi)(1 - t_\pi)(\xi_1 - 1) f/\delta f_e \right]^{1/k}$ (see Egger/Kreickemeier, 2009a for the same result).

### 3.3 Equilibrium (long-term) unemployment, aggregate output and the number of firms

In order to pin down the aggregate unemployment rate in the general equilibrium, we make use of the well-known concepts of wage-setting and price-setting schedules (see Layard et al., 1991). Consider first aggregate price-setting behavior. The representative firm chooses $p(\tilde{\phi}_t) = 1$. Then, the price rule (7) delivers the feasible real wage (henceforth FRW):

$$w_{PS}(\tilde{\phi}_t) = \frac{1}{1 + t_{pw}} \kappa a(\tilde{\phi}_t) \cdot \tilde{\phi}_t. \quad (47)$$
The FRW is independent of (un)employment, which is no surprise because of our assumptions about technology (output is linear to labor) and the constant price elasticity of product demand. As a specification of our model, the FRW depends positively on the average ability level. Observing $\phi_t = \Theta_t$ and (18) yields:

$$\pi = \Gamma_1 \Gamma_2 \Theta_t. \quad (48)$$

Let us turn to the target real wage. The (representative) monopoly union fixes the wage rate; we obtain (19). Taking the macroeconomic variables as given, the target real wage of the (representative) monopoly union can be written as:

$$w_{WS}(\Theta_t) = A^\omega/k \left(1 - u(1 - s)\right) \left(w(\Theta_t)\right)^{1-\omega} \cdot \Theta_t. \quad (49)$$

In the general equilibrium, we have $w_{PS}(\Theta_t) = w_{WS}(\Theta_t) = w(\Theta_t)$. With this condition, we can calculate the number of long-term unemployed $L^L$, the number of active workers $L$, the number of employed active workers $H$, the aggregate unemployment rate $\pi$, the aggregate output $Y$, the aggregate wage income $W$, the initial investment costs $Y_e$ and the number of firms $M$ for any given parameter setting of the government (see Appendix B):\[18\]

$$L^L = \left(1 - \xi_1^{\alpha k/\beta - \alpha k}\right) \frac{\pi}{\Gamma_3}, \quad (50)$$

$$L = \xi_1^{\alpha k/\beta - \alpha k} \cdot \frac{\pi}{\Gamma_3}. \quad (51)$$

$$H = \frac{\Gamma_3 \left(\Theta D\right)^{\omega} - s}{1 - s} \left(\xi_1^{\alpha k/\beta - \alpha k}\right) \cdot \frac{\pi}{\Gamma_3}. \quad (52)$$

$$\pi = 1 - \frac{H}{\Gamma_3}, \quad (53)$$

$$Y = \psi_2 \Gamma_1 \Gamma_2 \left(\Theta D\right)^{1+\alpha} H, \quad (54)$$

$$W = \kappa Y, \quad (55)$$

$$Y_e = M \pi^\text{net} = \Pi^\text{net}, \quad (56)$$

\[18\]The stability of the general equilibrium turns out to be critical in one way. Theoretically, the marginal firm has an incentive to deviate from the (monopoly union) wage setting in order to increase its profit. As explored in detail by de Pinto and Michaelis (2011), however, we can avoid this behavior by assuming a further labor market friction, i.e., efficiency wages. Clearly, extending the model in that way has a value added. But, balancing this value added with the loss of analytical tractability, we decided to postpone this issue to further research and to refrain from giving marginal firms additional latitude.
The definitions used are:

\[ M = \frac{Y}{(1 + \chi) \xi_1 D^\beta f} \]  \hspace{1cm} (57)

Inserting (52), (57) and (54) into (36) leads to the equilibrium number of employed workers by the representative firm, \( h(\bar{\phi}_1) \). Owing to (2), we can then determine \( q(\bar{\phi}_1) \).

### 3.4 Aggregate income and the Gini coefficient

Next, we determine overall income. By definition, overall income is the sum of aggregate wage income and aggregate profit income: \( I \equiv W + \Pi \) with \( \Pi = \Pi^{net} + T_x \). However, as a property of the underlying Melitz model, aggregate net profits are used to finance the initial investment costs [see (56)] and are not available for consumption spending. In common with the corresponding literature, we thus exclude \( \Pi^{net} \) from our overall income measurement, implying

\[ I = W + T_x. \]  \hspace{1cm} (58)

To compute the Gini coefficient \( \Theta \), we first have to determine the Lorenz curve (see Egger/Kreickemeier, 2012 for the general analytical approach). Therefore, we combine cumulative income with the proportion of individuals receiving this income. Recall that there are three sources of income for individuals in our model: \( B^l \), \( B^u \) and \( W^{net} \). Moreover, we have to be aware that a subgroup of firms engage in exporting and thus pay wages from a different profile than firms serving only the domestic market. Appendix C provides the step-by-step derivation of the Lorenz curve, while we only focus here on the main equations.

Let us start with the aggregate income of individuals with an ability level up to \( \tilde{a} \in [1, a^\star] \), i.e. long-term unemployed persons with \( B^l \) as income. Using (22), we can calculate the cumulative income of this subgroup as a proportion of overall income:

\[ \frac{I(\tilde{a})}{I} = \frac{1}{I} \int_1^{\tilde{a}} s(1 - t_w) w(\tilde{\phi}_1) g_a(a) u^T da. \]

Next, we define \( \eta \) as the proportion of the long-term unemployed persons on the total labor force, which is given by:

\[ \eta = \frac{U(\tilde{a})}{L} = \frac{1}{L} \int_1^{\tilde{a}} g_a(a) u^T da. \]
Putting together the distribution of the long-term unemployed persons with their income distribution, we obtain the first segment of the Lorenz curve:

\[ Q^1_L(\eta) = B^l \frac{T}{L} \quad \text{if } 0 \leq \eta \leq c_1, \quad (59) \]

where \( c_1 \equiv \eta(a^*) = 1 - (a^*)^{-k} \) denotes its boundary. \( Q^1_L(\eta) \) shows the share of income accruing to the lowest \( \eta \) percent of individuals in the income distribution. Evaluating \( Q^1_L(\eta) \) at \( c_1 \) and observing \( L^l = (1 - (a^*)^{-k}) T \) yields:

\[ Q^1_L(c_1) = B^l. \quad (60) \]

Considering, in addition, the aggregate income of unemployed individuals with an ability level up to \( \tilde{a} \in [a^*, \infty] \), i.e. \( B^u \), the ratio between the cumulative income of unemployed persons and overall income is given by:

\[ \frac{I(\tilde{a})}{I} = Q^1_L(c_1) + \frac{1}{I} \int_{a^*}^{\tilde{a}} s(1-t_w)a^\omega \left( w(\tilde{a}_u) \right)^{1-\omega} g_u(a)uLda. \]

Linking \( I(\tilde{a})/I \) with the proportion of individuals receiving this income:

\[ \eta \equiv \frac{U(\tilde{a})}{L} = \frac{U(a^*)}{L} + \frac{1}{I} \int_{a^*}^{\tilde{a}} g_u(a)uLda, \]

leads to the second segment of the Lorenz curve:

\[ Q^2_L(\eta) = \frac{1}{I} \left[ B^l + B^u \left( 1 - \left( 1 - \frac{T}{L^u} \left( \eta - \frac{L^l}{T} \right)^{k-1} \right) \right) \right] \]

\[ \text{if } c_1 \leq \eta \leq c_2, \quad (61) \]

where \( L^u \) represents the number of unemployed active workers. To compute its boundary, we have to consider all unemployed workers over the full distribution of abilities. Thus, we get \( c_2 \equiv \eta(\tilde{a} \to \infty) = (L^l + L^u)/L = \overline{u} \). Evaluating \( Q^2_L(\eta) \) at \( c_2 \) leads to:

\[ Q^2_L(c_2) = B \quad \text{and} \quad B \quad (62) \]

Now we add the income distribution of employed workers to the income distribution of unemployed individuals. Looking at firms serving the domestic market only, the ratio between the aggregate income of unemployed persons and of workers employed in firms with productivity levels up to \( \phi \in [\phi^*, \phi^*_2] \) and overall income is given by:

\[ \frac{I(\phi)}{I} = Q^2_L(c_2) + \frac{1}{I} \int_{\phi^*}^{\phi^*_2} \frac{w^{net}(\phi)}{\phi} h(\phi) M(\phi) d\phi. \]
Notably, the income of employed workers is equal to the net wage rate. The proportion of unemployed workers and workers employed in firms with productivity levels lower than or equal to $\phi$ on the total labor force is defined by:

$$\eta = \frac{U(\hat{\alpha} \rightarrow \infty) + H(\hat{\phi})}{L} = \pi + \frac{1}{L} \int_{\phi^*} \frac{h(\phi) M(\mu(\phi))}{\phi^*} d\phi.$$

Combining this expression with $I(\hat{\phi})/I$ yields the third segment of the Lorenz curve:

$$Q^3_L(\eta) = \frac{1}{I} \left[ B + \frac{W_{net}}{\Delta_1} \left( 1 - \left( 1 - \Delta_2 (\eta - \pi) \frac{T}{H} \right)^{\frac{k_\beta}{k_\sigma}} \right) \right]$$

if $c_2 \leq \eta \leq c_3$.

c_3 represents the boundary of the third segment with $\hat{\phi} = \phi_3^*$. Using (27) implies:

$$c_3 \equiv \eta \left( \phi = \phi_3^* \right) = \frac{U(\hat{\alpha} \rightarrow \infty) + H(\hat{\phi}_3^*)}{L} = \pi + \frac{H}{L} \Delta_2 \left( 1 - \chi^{\frac{k_\beta + \alpha \sigma}{k_\sigma}} \right).$$

Evaluating $Q^3_L(\eta)$ at $c_3$ yields:

$$Q^3_L(c_3) = \frac{1}{I} \left[ B + \frac{W_{net}}{\Delta_1} \left( 1 - \chi^{\frac{k_\beta}{k_\sigma}} \right) \right].$$

In the last step of the Lorenz curve derivation, we additionally consider workers who are employed in firms with productivity levels up to $\hat{\phi} = [\phi_3^*, \infty]$, i.e. exporters, for the income distribution:

$$I(\hat{\phi}) = Q^3_L(c_3) + \frac{1}{I} \int_{\phi_3^*}^{\infty} W_{net}(\phi) h_t(\phi) M_x(\mu_x(\phi)) d\phi,$$

with $h_t(\phi) = \Delta_3 h(\phi)$ and $\Delta_3 \equiv 1 + \tau^{1-\sigma}$. Linking this expression with the proportion of workers who are unemployed and employed in firms with productivity levels lower than or equal to $\hat{\phi}$:

$$\eta = \frac{U(\hat{\alpha} \rightarrow \infty) + H(\hat{\phi})}{L} = \pi + \frac{H(\phi_3^*)}{L} + \frac{1}{L} \int_{\phi_3^*}^{\infty} h_t(\phi) M_x(\mu_x(\phi)) d\phi,$$

yields the fourth segment of the Lorenz curve:

$$Q^4_L(\eta) = \frac{1}{I} \left[ B + W_{net} \left( 1 - \Delta_3 \left( 1 - (\eta - \pi) \frac{T}{H} \right)^{\frac{\Delta_2}{\Delta_3}} \right)^{\frac{k_\beta}{k_\sigma}} \right].$$

(65)
if $c_3 \leq \eta \leq 1$.

Combining the four segments, we can write the Lorenz curve as:

$$Q_T = \begin{cases} 
Q^L_1(\eta) & \text{if } 0 \leq \eta \leq c_1 \\
Q^L_2(\eta) & \text{if } c_1 \leq \eta \leq c_2 \\
Q^L_3(\eta) & \text{if } c_2 \leq \eta \leq c_3 \\
Q^L_4(\eta) & \text{if } c_3 \leq \eta \leq 1
\end{cases} \quad (66)$$

With the Lorenz curve at hand, we can compute the Gini coefficient from

$$\Theta = 1 - 2 \int_0^1 Q_T(\eta) \, d\eta. \quad (67)$$

### 3.5 The government’s optimization problem

Given the outcomes of the general equilibrium, we are now able to specify the government’s optimization problem. In general, the government seeks for an RS which maximizes its objective function (25) subject to the budget constraint. Therefore, two subsequent decisions has to make. Recall that the expenditure side is described by UB only, while there are three kind of taxes characterizing the revenue side.

In the first step, the government chooses one of the three types of tax to finance UB.\(^{19}\) Thus, we can distinguish between RS 1 (UB financed by a wage tax), RS 2 (UB financed by a payroll tax) and RS 3 (UB financed by a profit tax paid by the exporters) which are each connected with a separate budget constraint, respectively:

$$B = T_w = t'_w W. \quad (68)$$

$$B = T_{pw} = t'_{pw} W. \quad (69)$$

$$B = T_\pi. \quad (70)$$

Clearly, the government’s policy instruments are not revenue-neutral but have repercussion effects on the budget. To avoid further complications from this channel, we follow Creedy and McDonald (1992) as well as Goerke (1996) in assuming that the budget is ex ante revenue-neutral, i.e. the budget does not vary in response to the government’s policy. We indicate the corresponding tax rates with an apostrophe.\(^{20}\) Note that we can easily compute the budget

\(^{19}\)Note that we abstain from mixing the three sources of income in order to consider the diverging effects of the differential taxes separately.

\(^{20}\)For a general equilibrium model with ex post revenue-neutrality, i.e. one in which the budget is neutral after the consideration of all possible adjustments in the economy, see Michaelis and Pflüger (2000).
constraints explicitly because the general equilibrium pins down the required variables (see de Pinto, 2012).

In the second step, the government sets the replacement ratio \( s \), which determines the level of UB, in order to maximize welfare under consideration of the respective budget constraint. Formally, we obtain from (25):

\[
\max_s W F_{RS_1} = \frac{I_{RS_1}}{L} \Theta_{RS_1}^{-\gamma} \quad s.t. \quad (68), \quad (t_{pw} = t_{\pi} = 0),
\]

\[
\max_s W F_{RS_2} = \frac{I_{RS_2}}{L} \Theta_{RS_2}^{-\gamma} \quad s.t. \quad (69), \quad (t_{w} = t_{\pi} = 0),
\]

\[
\max_s W F_{RS_3} = \frac{I_{RS_3}}{L} \Theta_{RS_3}^{-\gamma} \quad s.t. \quad (70), \quad (t_{w} = t_{pw} = 0).
\]

We call the solutions of three separate maximization problems local optima, i.e. the welfare maximum for each of the three RS. Using (58) and (67) as well as the outcomes of the general equilibrium, the local optima can be calculated.\(^{21}\) Once the local optima are determined, the government find the global optimum by simply comparing the welfare maxima of the three RS. We call the corresponding combination of UB and tax form optimal RS.

4 The redistribution schemes

4.1 Calibration

Now we solve the government’s optimization problem for the corresponding RS. Analytically, we could reformulate \( I \) and \( \Theta \) as functions depending only on the model parameters and on the policy variable \( s \). Thus, a closed form solution of the optimal value of \( s \) is possible. However, the degree of complexity is substantial, in particular because of the Gini coefficient’s derivation, which restricts the explanatory power of the model’s outcome. In order to obtain explicit results, we thus simulate our model. The following numerical illustration is based on standard practice in the literature. Table 1 summarizes the parameter values for monthly time periods.

Table 1 about here

We follow Ghironi and Melitz (2005) as well as Felbermayr et al. (2011) to calibrate most of the Melitz model elements, but we make one substantial variation. As stated by Eaton et al. (2004), we set the shape parameter of the Pareto distribution to be equal to 4.2, which is relatively higher in comparison to the literature.
to its standard calibration value of 3.4. This variation can be justified by the nature of the general equilibrium without government’ activities. Observing (52) and $u = 1 - H/L$ shows that $\Gamma \cdot \phi^\omega \leq 1$ must hold to ensure $0 \leq u \leq 1$.22

Put differently, the aggregate labor demand $H$ must not exceed the number of active workers $L$ in the equilibrium. This condition is c.p. fulfilled if the shape parameter $k$ is sufficiently high. The reason for this is simple. The higher $k$, the larger is the fraction of firms with an entrepreneurial productivity close to the cut-off level, the larger is the fraction of firms with a relatively low minimum quality requirement, and the larger is the number of active workers. Thus, our slightly different calibration with $k = 4.2$ is needed to guarantee the existence of an equilibrium at the starting position, without offending against the empirical findings.

Three other parameters are specific to our approach, namely $\omega$, $\alpha$, and $\gamma$. The parameter $\omega$, measuring the weight of the abilities in the wage determination, has only been estimated in a few studies. Keane (1993) claims that 84 percent of wage differences across industries are explained by individual fixed effects, while only 16 percent can be traced back to industry dummies. The strong weight of individual characteristics in the wage determination is confirmed by, for instance, Fairris and Jonasson (2008) and Holmlund and Zetterberg (1991). Hence, a value of $\omega = 0.8$ does not seem at odds with the empirical literature. Unfortunately, to the best of our knowledge, there is no empirical estimation for the parameter $\alpha$, which captures the strength of the minimum quality requirements. Intuitively, $\alpha$ should be smaller than 1. We set $\alpha = 0.25$, implying that the minimum quality requirement is relatively weak. Thus, the quality of the firm’s management, $\phi$, is significant higher than is the ability level of its least efficient worker, $\phi^\omega$. In our opinion, this should be the case in nearly all firms; nevertheless, an empirical estimation of $\alpha$ is a task for future research. Concerning the parameter of inequality aversion, $\gamma$, which can be interpreted as inequality elasticity (see section 2.5), there is relatively good empirical evidence. Carlsson et al. (2005) use an experimental design to show that the value of inequality aversion is in the interval between 0.09 and 0.22. Schwarze and Härpfer (2007) measure income inequality by an empirical analysis with data from the GSOEP from 1985 to 1998 and obtain a value about 0.2. Thus, we set $\gamma = 0.2$. Additionally, we normalize the price index and total labor force to one without any loss of generality.

4.2 Local optima

Before we look at the local optima, let us first briefly consider the impact of trade liberalization (for a detailed discussion, see de Pinto, 2012). Given the parameter setting above, trade liberalization, e.g., a reduction of variable trade costs from $\tau_0 = 1.6$ to $\tau_1 = 1.3$, unambiguously increases aggregate income because of the well-known firm-selection effect. Furthermore, it increases income inequality because of both the increasing number of exporters who pay higher...

22For a similar problem, see Egger and Kreickemeier (2009a).
wages to a higher share of workers and the increasing unemployment rate. Both
increasing aggregate income and a more unequal distribution of income motivate
the government’s redistribution aims.

We present the simulation results for the RS in the following manner. In
each figure, we show the reaction of the two objective functions’ arguments, \( I \)
and \( \Theta \), to variations in \( s \). Furthermore, we illustrate the \( s \)-welfare relation as a
consequence of the evolution of \( I \) and \( \Theta \). The starting point \(( s = 0)\) indicates the
level of the dependent variable in a trade liberalized economy without market
intervention by the government, we plot this reference level by the dotted black
line. A second reference point, the values of \( I, \Theta \) and welfare before trade
liberalization, is illustrated by the continuous black line.

4.2.1 RS 1: UB financed by the wage tax

Let us start with RS 1, where UB are financed by a wage tax. Figure 1 illustrates
the simulation results.

First, overall income monotonically decreases in \( s \). According to (58) and (55),
aggregate output \( Y \) is responsible for the variation in \( I_{RS1} \). There are two
channels through which \( s \) and thus UB influence \( Y \). First, UB enhance the
monopoly unions’ target real wage at any given level of employment because
of an increasing fallback income of the median member [see (49)]. The firm’s
answer to such a rise in its marginal costs is an increase in its profit-maximizing
price. Product and labor demand drop, and the number of employed workers
decreases [see (52)]. The decline in \( H \) leads to a reduction in the overall firm’s
production; the aggregate output \( Y \) shrinks. We call this channel the “UB-effect
on \( Y \)” in the following. Second, we have to take into account the consequences
of the UB funding, i.e. here the wage tax. However, as pointed out in more
detail by de Pinto (2012), the wage tax is neutral for the aggregate variables
at the general equilibrium. The target real wage does not vary because of the
increasing wage mark-up [see (9)] and the decreasing fallback income [see (13)],
which exactly offset each other.\(^{23}\) Furthermore, the decline in \( b \) would expand
the firm-specific interval of abilities, but, because the net wage rate decreases by
the same magnitude, \( a_z \) is unaffected. Both results imply wage tax neutrality.

Turning to the Gini coefficient, we find a u-shaped reaction. There are two
effects operating in opposite directions. On the one hand, paying UB reduces
income inequality because so far unemployed workers’ income increases. On
the other hand, UB raises the aggregate unemployment rate \( \pi \) [see (53)]. The
proportion of workers receiving a relatively low income increases and hence
income inequality rises. As shown in Figure 1, the former effect dominates the

\(^{23}\)Notably, this finding strongly depends on the assumption of using the net outside wage
in the computation of UB [see (12)]. If instead \( B_j = s \pi J \) is applied, the decline in fallback
income becomes smaller and thus it does not compensate the increasing wage claim – \( w \)
would be a positive function of \( t_w \). However, simulations show that a variation in the wage
tax rate has an extremely low influence on \( w \). Thus, we ignore this effect in the following.
The corresponding simulation results are available upon request.
latter for relatively low values of \( s \), whereas income inequality rises for relatively high values of \( s \). We call this mechanism the “UB-effect on \( \Theta \)” in the following. Note that due to wage tax neutrality, \( t_w \) has no impact on the Gini coefficient.

Putting this together, we observe a hump-shaped welfare reaction. Starting at \( s = 0 \), welfare increases in \( s \) since the UB-effect on \( \Theta \) generates a reduction of income inequality that is high enough to overcompensate the decline in overall income caused by the UB-effect on \( Y \). At the welfare maximum, both effects exactly offset each other; after this, the decline in \( Y \) dominates the reduction in income inequality. The welfare reduction becomes even stronger (the curve becomes steeper) when the UB-effect on \( \Theta \) produces an increase in income inequality. As a result, we find the local optimum at \( s = s^*_{RS_1} \) which pins down the welfare-maximizing combination of \( I_{RS_1} \) and \( \Theta_{RS_1}^* \).

**Proposition 1** Suppose that the government chooses RS 1 - UB are financed by a wage tax. Then, (i) overall income unambiguously decreases in \( s \), (ii) the income inequality reaction is u-shaped and (iii) the welfare reaction is hump-shaped, achieving a local optimum at \( s = s^*_{RS_1} \). At the local optimum, the welfare-maximizing combination of \( I_{RS_1} \) and \( \Theta_{RS_1}^* \) is determined.

**Proof.** see text and Figure 1.

Note that the wage tax neutrality is an analytical result which holds for any parameter constellation. In the same way, the sign of the UB-effect on \( Y \) and the u-shaped UB-effect on \( \Theta \) are also independent of the chosen parameter setting. Thus, the existence of the local optimum is robust, but of course, the location of it varies with the model’s parameters.

4.2.2 RS 2: UB financed by the payroll tax

Next, we consider RS 2 where UB are financed by a payroll tax. Figure 2 illustrates the simulation results.

Overall income monotonically declines, which is a reflection of the aggregate output reduction [see (58) and (55)]. We observe two channels. First, the UB-effect on \( Y \) occurs; aggregate output declines. Second, the payroll tax increases the firm’s marginal costs. As a consequence, the profit-maximizing price rises, leading to a decline in product demand, revenues and profits. The number of firms operating in the market shrinks and thus aggregate employment declines. The reduction of \( H \) leads to a one-to-one decrease in aggregate output [see (54)].

Looking at the income distribution, we observe an increase in \( \Theta \) for relatively low values of \( s \), while \( \Theta \) decreases for relatively high values of \( s \). The economic intuition behind this result is based on the impact of the payroll tax on the

\[24\] Note that the different impact between the wage and the payroll tax is caused by diverging influences of both on the fallback income [see (13) and (47)]. This result is well-documented in the literature; see for instance Goerke (2002).
goods and labor markets. We know from earlier statements that firms increase their prices if \( t_{pw} \) rises, which leads to a decline in the FRW [see (47)]. On the macroeconomic level, the equilibrium wage rate, \( w(\hat{\theta}_t) \), falls. The reduction in \( w(\hat{\theta}_t) \) passes through to wage setting at the sectoral level because of the implied decrease in the outside wage [see (11)]. Thus, unions set c.p. a lower wage rate, which causes a decline in the wage rate paid by all firms operating in the market. If \( w \) decreases, the number of workers employed by active firms rises, but there is also the decline in goods demand because of the payroll tax mentioned above, which reduces \( h \). In the equilibrium, the two effects exactly offset each other; the employment per firm remains constant. Note, however, that the number of firms declines, as explored above, which causes a reduction in \( H \).

With this reasoning, we can explain the hump-shaped \( \Theta \). On the one hand, \( H \) falls and conversely \( \pi \) rises, which increases income inequality. On the other hand, the income gap between UB and wage income shrinks because of the increase in \( s \) and the decline in \( w \); income inequality decreases. Clearly, this effect becomes stronger the higher \( t_{pw} \) or the lower \( w \) is. It is evidently that the former effect dominates for relatively low values of \( t_{pw} \), while the latter dominates for relatively high values of \( t_{pw} \). Note that the UB-effect on \( \Theta \) also occurs. However, as seen in Figure 2, the derived payroll tax influence on \( \Theta \) turns the u-shaped UB-effect (see Figure 1) into a hump-shaped reaction for the Gini coefficient.

Welfare unambiguously declines in \( s \). If UB are low, welfare decreases because of both the reduction of \( I \) and the increase in \( \Theta \). If UB are high, then income inequality drops, which mitigates but does not compensate for the decline in overall income; welfare still shrinks. As a result, \( s_{RS_2}^* = 0 \) determines the local optimum. The welfare-maximizing combination of \( I_{RS_2} \) and \( \Theta_{RS_2} \) is identical with the corresponding level at the starting position.

**Proposition 2** Suppose that the government chooses RS 2 – UB are financed by a payroll tax. Then, (i) overall income unambiguously decreases in \( s \), (ii) the income inequality reaction is hump-shaped and (iii) welfare unambiguously declines; the local optimum is given by \( s_{RS_2}^* = 0 \).

**Proof.** see text and Figure 2. ■

Looking at the robustness of this result, the negative effect of the payroll tax on aggregate income is parameter-independent. Combining with UB, the reduction of aggregate income is robust. The qualitative impact of the payroll tax on income inequality, i.e. the hump-shaped reaction, does not vary with the parameter-setting, but the quantitative results change. Thus, mixing the payroll tax channel and the u-shaped UB-effect on \( \Theta \) produces different outcomes for different parameter choices. Since the effects are opposite, however, the variation is relatively small. Consequently, the local optimum \( s_{RS_2}^* = 0 \) is relatively robust; in some special cases, \( s_{RS_2}^* \) is larger but very close to zero.
4.2.3 RS 3: UB financed by the profit tax

In RS 3, UB are financed by a profit tax paid exclusively by exporters. In our simulation results, which are illustrated in Figure 3, we additionally look at the special case where the profit tax is paid by all firms (see blue dotted line). Throughout our analysis, we focus on the former but additionally explain the key differences between both cases.

At first, we should make one important remark. Due to the export cut-off condition (31), the profit tax rate has to be smaller than $t_{n}^{\text{max}} = \frac{1}{1-\sigma}$. If $t_{n}$ exceeds this threshold, no firm, independent of its entrepreneurial productivity, has an incentive to export – tax revenue and UB would be zero. Moreover, we also see from (31) that if $t_{n}$ converges to $t_{n}^{\text{max}}$, the ratio between $\tilde{\phi}_{x}$ and $\tilde{\phi}$ increases exponentially. Using (70), our simulation indicates that for $s < s^{\text{critical}} = 0.25$, $t_{n}$ is sufficiently lower than $t_{n}^{\text{max}}$ to avoid $\tilde{\phi}_{x} \gg \tilde{\phi}$ and complications from this unrealistic setting.

As shown in Figure 3, overall income decreases, which is again related to the decline in aggregate output [see (58) and (55)]. We can explain the reduction of $Y$ by the interplay of three effects. First, we observe the UB-effect on $Y$, reducing aggregate output and overall income. Second, the profit tax implies a decrease in the cut-off productivity. To prove that, we simulate Eq. (44), resulting in $\frac{\partial \pi_{n}^{\text{net}}}{\partial t_{n}} < 0$. Consequently, the present value of average net profits $(1 - G(\phi^{*}))\pi_{n}^{\text{net}}/\delta$ decreases for any given level of $\phi^{*}$. Hence, the entry into the Melitz lottery is less attractive, which c.p. reduces the number of firms passing through the lottery successfully. Thus, the number of available goods in the market, $M_{t}$, shrinks, implying c.p. an increased demand for each variety [see (1)]. Consequently, the revenues of all firms shift up so that firms that are less productive than before the profit tax was introduced can cover their fixed costs and enter the market; $\phi^{*}$ decreases (see de Pinto, 2012 for the same result). Consequently, the distribution of active firms shifts down, which has two important implications. On the one hand, there are more firms demanding low-skilled workers, which reduces the number of long-term unemployed persons [see (50)]; aggregate employment and output increase. On the other hand, the average productivity of all domestic firms, $\tilde{\phi}$, falls [see (28)]. The reduction in $\tilde{\phi}$ yields a decline in the average productivity of all firms which are active in the market, $\phi_{t}$ [see (35)]. As a result, marginal costs increase, leading to a reduction of the FRW and forcing down the employment level $H$ as well as the aggregate output $Y$. Third, we find the ES $(D > 1)$ if $t_{n} > 0$. Total average productivity increases [see (35)], the FRW shifts up and consequently $H$ and $Y$ both rise.

It is evident from Figure 3 that the negative effects due to UB and the decreasing $\tilde{\phi}$ dominate the positive effects of the lower number of long-term unemployed persons and ES; overall income declines. Moreover, if the profit tax is paid by all firms, $\phi_{t}^{*}$ is independent of $t_{n}$. Thus, ES and therefore one of the positive channels vanish, while the others remain; the decline in overall income becomes stronger.
Turning to income distribution, we find that the Gini coefficient declines in $s$. Again, the UB-effect on $\Theta$ appears, but recall that we only consider constellations for $s \leq s_{\text{critical}} = 0.25$. In this parameter range, the UB-effect causes a decline in $\Theta$ (see Figure 1). Additionally, this channel interacts with the consequences of the profit tax for income inequality. To analyze their implications in more detail, we simulate Eq. (52) and obtain $\partial H/\partial t_\tau > 0$. Thus, the combined positive effects of lower long-term unemployment and higher ES dominate the negative effect of the lower $\phi$; employment unambiguously increases. The decrease in unemployment decreases income inequality; $\Theta$ declines. Moreover, the composition of firms varies. If $t_\tau$ rises, $\phi^*_x$ shifts up and the number of exporting firms decreases. As a consequence, the employment share of high-wage firms, i.e. exporters, goes down, leading to lower inequality among employed workers which results in a reduction of $\Theta$. Summing up, UB financed by a profit tax unambiguously forces down income inequality. In the case where the profit tax is paid by all firms, the reduction is smaller because both ES and the composition effect disappear.

Finally, the interplay of overall income and variation in income inequality implies a hump-shaped reaction for welfare. If $s$ is relatively small, the decline in the Gini coefficient dominates the decline in overall income; welfare increases. If $s$ is relatively high, the reverse conclusion holds; welfare decreases. The local optimum is achieved at $s = s_{RS}$. Interestingly, the result changes substantially if the profit tax is paid by all firms. Without ES and the composition effect, the decline in overall income becomes stronger, while the decline in income inequality is mitigated, which yields a monotonically welfare reduction. Then, the government has no opportunity to improve welfare with the RS and the optimal value of $s$ would be zero.

**Proposition 3** Suppose that the government chooses RS 3 – UB are financed by a profit tax paid by exporters. Then, (i) overall income unambiguously decreases in $s$, (ii) income inequality unambiguously decreases and (iii) the welfare reaction is hump-shaped, achieving a local optimum at $s = s_{RS}^*$. At the local optimum, the welfare-maximizing combination of $I_{RS3}^*$ and $\Theta_{RS3}^*$ is determined. **Proof.** see text and Figure 3.

In the case of RS 3, our findings are parameter-dependent. As mentioned, there are different effects operating in opposite directions. Which effect dominates is a question of the parameters chosen. Varying the parameters shows that the decline in aggregate income is robust. However, the reduction of income inequality can be mitigated, for example if $\omega$ increases. In a few extreme scenarios, this even implies an unambiguous decline in welfare for all $s$. Notably, the parameter choice is then very different from the standard practice in the literature. In contrast, our results concerning an RS where UB are financed by a profit tax paid by all firms are relatively robust because ES disappears. Furthermore, the relative disadvantage (in terms of welfare) of a profit tax paid by all firms compared to the former case remains for all parameter-settings.
4.3 Global optimum

After deriving the local optimum for each RS, we derive the optimal RS. From our previous statements, it is evident that the government will choose neither a payroll tax nor a profit tax paid by all firms to finance UB, because this would decrease welfare immediately. Thus, the decision between RS 1 and RS 3, which are illustrated in Figure 4 by the brown line and the blue line respectively, is the relevant one.

Looking at Figure 4, we find that the local optimum of RS 3 determines the global welfare maximum. The optimal RS is to choose a profit tax paid by exporters only and to set $s = s_{RS3}$.

**Proposition 4** Suppose that the government seeks the optimal RS that maximizes welfare. Due to $WF_{RS3}(s_{RS3}) > WF_{RS1}(s_{RS1}) > WF_{RS2}$, the optimal value of UB is given by $s = s_{RS3}$ and the optimal financial form is the profit tax paid by exporters.

**Proof:** see Figure 4. ■

Note that this result is relatively robust if the parameter for inequality aversion, $\gamma$, varies. Only if $\gamma$ drops close to zero, the optimal RS would be to finance UB by the wage tax. In the special case of $\gamma = 0$, the model collapses to the standard case where welfare depends only on aggregate income implying that the equity-efficiency trade-off would be disregarded. Then, the government can only maximize the level of equity (measured by the value of the UB) under the condition that aggregate income does not fall below its benchmark level before trade liberalization (see de Pinto, 2012, for a detailed discussion of this model variant).

5 Conclusion

The contribution of this paper is to derive the government’s optimal RS for trade gains in the case of a government’s objective function which considers both aggregate income and income distribution and therefore accounts explicitly for the equity-efficiency trade-off. Three RS are distinguished: first, the government pays UB financed by a wage tax. Second, the government imposes a payroll tax for its funding and third, the government finances UB by a profit tax that is exclusively paid by exporting firms.

Using a Melitz type model of international trade with unionized labor markets and heterogeneous workers, we calculate the welfare maximizing threshold level of UB within each of the three RS, i.e. the local optimum. In the case of the payroll tax funding (RS 2), the optimal value of UB is zero because of welfare unambiguously decreases. If UB are financed by the wage tax (RS 1), there is a local optimum with a positive value of UB. While the wage tax is neutral, UB decrease aggregate income, but, for sufficiently low values of $s$, income inequality decreases and this dominates the former until the local optimum is reached.
If UB are financed by a profit tax paid by exporters (RS 3), this also generates a local optimum with UB different from zero. The key mechanism is that the profit tax unambiguously decreases income inequality, which overcompensates the decrease in aggregate income.

As a result, RS 1 and 3 Pareto dominate RS 2. Moreover, the global welfare maximum is given by the local optimum of RS 3. For our chosen parameter Settings, we thus find a ranking of the RS in terms of welfare level: 1. UB financed by a profit tax paid by exporters, 2. UB financed by a wage tax and 3. UB financed by a payroll tax. Obviously, our predictions are limited, in the sense that they only look at UB as a redistribution instrument. In future research, our approach can be extended to a policy mix, where in addition to UB we also consider for example employment subsidies, wage subsidies or progressive tax forms.

6 Appendix

Appendix A: Government sector

Using (22), we calculate the aggregate UB of the long-term unemployed persons:

$$B_l = s(1 - t_w)w(\bar{\phi}_l)uL_l.$$  \hfill (A1)

The aggregate UB of unemployed active workers is given by:

$$B^u = \int_{a^*}^{\infty} s(1 - t_w)a^{\omega}(w(\bar{\phi}_l))^{1-\omega}g_\alpha(a)uLda.$$  \hfill (A2)

Observing the Pareto distribution and $L = (a^*)^{-k}L$, we can solve the integral to get:

$$B^u = s(1 - t_w)\xi_3 \cdot (a^*)^{\omega}w(\bar{\phi}_l)^{1-\omega}uL,$$  \hfill (A3)

with $\xi_3 \equiv k/(k - \omega)$. Using $B = B^l + B^u$ and $u^l = 1$, we obtain the aggregate UB (38).

The wage tax and payroll tax use the aggregated wage income as a tax base, which is a constant share $\kappa$ of total output because of the mark-up pricing rule. We immediately get the aggregate tax revenues (39) and (40). The aggregate profit tax revenue is given by:

$$T_\pi = t_\pi \left( \int_{\phi_x}^{\infty} \pi(\phi) M_x \mu_x(\phi) d\phi + \int_{\phi_x}^{\infty} \pi_x(\phi) M_x \mu_x(\phi) d\phi \right).$$

Reformulating (20) for gross profits, noting $\phi_i = \phi$, $\pi(\phi) = r(\phi)/\sigma - f$ as well
as (23) implies:

\[ T_\pi = t_\pi \left( \frac{r(\bar{\phi}_x)}{\sigma} \bar{\phi}_x^{-\beta} M_x \left( \int_0^{\infty} \phi^\beta \mu_x(\phi) d\phi + \tau^{1-\sigma} \int_0^{\infty} \phi^\beta \mu_x(\phi) d\phi \right) - 2fM_x \right). \]

(A3)

As shown by Egger and Kreickemeier (2009b), the general solution of (29) is given by:

\[ \bar{\phi}_x = \left[ \int_0^{\infty} \phi^\beta \mu_x(\phi) d\phi \right]^{1/\beta}. \]

(A4)

Combining (A3) and (A4) leads to (41).

**Appendix B: Derivation of the general equilibrium**

For the number of long-term unemployed persons, we use \( P(a < a^*) = 1 - (a^*)^{-k} \) to obtain \( L' = (1 - (a^*)^{-k})L \). Observing (21) and (28) yields the number of long-term unemployed persons. Using \( L = L' - L_1 \), we get the number of active workers.

To calculate the employment, we combine (47) and (49) to eliminate the wage. This leads to

\[ \frac{1}{1+t_w} \kappa \bar{\phi}_t = A^{1/k}(1 - u(1 - s))^{1/w} \cdot \bar{\phi}_t. \]

(B1)

The substituting of (48) into (B1) and rearrangement leads at first to the unemployment rate of active workers \( u \). Inserting this result and (51) into \( H = (1 - u)L \) yields the number of employed active workers. By substituting (21), (28) and (51) into (37), we obtain \( \pi \).

Concerning aggregate output, we use (2) and (36) to obtain \( Y = M_t q(\bar{\phi}_t) = M_t h(\bar{\phi}_t) \pi(\bar{\phi}_t) \cdot \bar{\phi}_t = M_t \frac{\pi(\bar{\phi}_t)}{\frac{\xi_1}{M_t} + \xi_2 \psi} \cdot H \). Observing (48) and (35), we get aggregate output. As mentioned above, aggregate wage income is a constant share of aggregate output due to mark-up pricing.

Turning to the determination of the initial investment costs, we first observe \( Y_e = f_x M_e \), where \( M_e \) denotes the mass of firms participating in the Melitz lottery. In a stationary equilibrium, firms that are hit by the exogenous death shock have to be replaced by firms entering the market – those firms which pass the Melitz lottery successfully: \( \delta M = (1 - G(\phi^*))M_e = (\phi^*)^{-k} M_e \). Using (45) leads to \( M_e = M^{\text{net}}/f_x \). Inserting the latter into \( Y_e = f_x M_e \) and observing \( \pi^{\text{net}} = \Pi^{\text{net}} / M \) determines the initial investment cost.

Finally, we use \( Y = R = (1 + \chi)M r(\bar{\phi}_t) \) to calculate the number of firms \( M \).

Using \( r(\bar{\phi}_t) = r(\bar{\phi}_t) / \sigma - f \) and \( \pi(\bar{\phi}_t) = (D^\beta \xi_1 - 1)f \), we obtain \( r(\bar{\phi}_t) = D^\beta \xi_1 f \sigma \), which leads to the number of firms operating in the market.

**Appendix C: Derivation of the Lorenz curve**

First segment: Owing to the Pareto distribution, \( u^t \rightarrow 1 \) and (A1), we can reformulate the conditional equation for \( I(\bar{\alpha})/I \) with \( \bar{\alpha} \in [1, a^*) \) to

33
\[ \frac{I(\hat{a})}{I} = \left(1 - \frac{\hat{a}^{-k}}{\hat{a}^*} \right) \frac{B^{l'}}{T} \frac{T}{I} \]  
if \( \hat{a} < a^* \).  

(C1)

The proportion of the long-term unemployed persons on the total labor force, \( \eta \), for \( \hat{a} \in [1, a^*] \), drops to \( \eta = 1 - \hat{a}^{-k} \) due to the Pareto distribution. Inserting this expression into (C1) leads to (59).

Second segment: Inserting (A2), \( L = (a^*)^{-k} \bar{T} \) and (60) into \( I(\hat{a})/I \) for \( \hat{a} \in [a^*, \infty) \) as well as using the Pareto distribution yields:

\[ \frac{I(\hat{a})}{I} = \frac{1}{I} \left[ B^l + B^u \left(1 - \left(\frac{\hat{a}^*}{\hat{a}}\right)^{\omega-k}\right) \right] \]  
if \( \hat{a} \geq a^* \).  

(C2)

Substituting \( c_1 = 1 - (a^*)^{-k} \), \( L^l = (1 - (a^*)^{-k}) \bar{T} \) and \( L^u = u(a^*)^{-k} \bar{T} \) into \( \eta \) for \( \hat{a} \in [1, \infty] \) implies:

\[ \eta = \frac{1}{L} \left[ L^l + L^u \left(\frac{\hat{a}^*}{\hat{a}}\right)^{-k}\right] \]  
if \( \hat{a} > a^* \).  

(C3)

Solving (C3) for \( \hat{a}/a^* \) and inserting the result into (C2) leads to (61).

Third segment: Using \( w(\phi) h(\phi) = \kappa r(\phi) \), (32), (23), \( M_t = (1 + \chi) M \), (55), \( Y = M_t r(\phi_t) \), (35) and (28) as well as (62), we can solve \( I(\hat{\phi})/I \) for \( \hat{\phi} \in [\phi^*, \phi_x^*] \) to get:

\[ \frac{I(\hat{\phi})}{I} = \frac{1}{I} \left[ B + \frac{W^{net}}{\Delta_1} \left(1 - \left(\frac{\hat{\phi}}{\phi^*}\right)^{\beta-k}\right) \right] \]  
if \( \hat{\phi} < \phi_x^* \).  

(C4)

Subsequently inserting (32), (24), (36), (35) and (28) into \( \eta \) for \( \hat{\phi} \in [\phi^*, \phi_x^*] \) leads to:

\[ \eta = \bar{\pi} + \frac{H}{L} \frac{1}{\Delta_2} \left(1 - \left(\frac{\hat{\phi}}{\phi_x^*}\right)^{\beta-k}\right) \]  
if \( \hat{\phi} < \phi_x^* \).  

(C5)

Solving (C5) for \( \hat{\phi}/\phi^* \) and substituting the result into (C4) yields (63).

Fourth segment: Using the same analytical approach as before and observing (26'), (34), \( M_t = (1 + \chi) M \), \( M_x = \chi M \) as well as (64), we can rewrite \( I(\hat{\phi})/I \) for \( \hat{\phi} \in [\phi_x^*, \infty) \) to obtain:

\[ \frac{I(\hat{\phi})}{I} = \frac{1}{I} \left[ B + \frac{W^{net}}{\Delta_1} \left(1 - \frac{\Delta_3}{\Delta_1} \left(\frac{\hat{\phi}}{\phi_x^*}\right)^{\beta-k}\right) \right] \]  
if \( \hat{\phi} \geq \phi_x^* \).  

(C6)

By analogy, we can solve \( \eta \) for \( \hat{\phi} \in [\phi_x^*, \infty] \) which results in:

\[ \eta = \bar{\pi} + \frac{H}{L} \left(1 - \frac{\Delta_3}{\Delta_2} \left(\frac{\hat{\phi}}{\phi_x^*}\right)^{\beta-k}\right) \]  
if \( \hat{\phi} > \phi_x^* \).  

(C7)

Combining (C7) and (C6) in the usual manner implies (65).
References


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Table 1: Calibration

Figure 1: UB and the wage tax
Figure 2: UB and the payroll tax

Figure 3: UB and the profit tax
Figure 4: Optimal RS
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